STM Studio

General N-Face Nodal Construction, Checking & Stress Reconstruction for STM Models

Document Revision

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Abstract

This paper presents the formulation and current implementation adopted in RKALC Structural Engineering's STM Studio for the construction, checking, assessment and visualisation of general two-dimensional nodal zones in strut-and-tie models. The workflow deliberately separates the solved analytical STM from the finite physical nodal geometry. An equilibrated set of analytical nodal actions is used to initialise a convex polygon, while physical load and support contacts are treated as authoritative fixed faces. The engineer may then graphically edit the remaining faces, merge adjacent analytical faces into a single physical face, and freeze the geometry independently of whether the current nodal strength check passes. A member face may also be absorbed into an adjacent load or support face without changing that external contact's position, angle or width; the retained physical face carries the vector sum of its assigned analytical actions and the constituent offset members terminate on that same face. Each physical face resultant is resolved relative to the actual face into normal and tangential traction components. A whole-node volume-averaged Cauchy stress tensor is recovered directly from the boundary resultants and face-centre position vectors, followed by principal stresses, maximum shear and principal directions. Because a single uniform tensor cannot generally reproduce all boundary tractions, STM Studio reports a traction-compatibility diagnostic and constructs a local centroid-fan constant-strain-triangle (CST) elastic equilibrium reconstruction for heat-map visualisation. In parallel, the present STM Studio implementation provides a separate nodal strength layer: three-action nodes are classified from the underlying analytical actions as CCC, CCT or CTT for automatic AS 3600:2018 checking, whereas TTT and more complex nodes require a designer-entered allowable stress. The local CST field remains an interpretation and diagnostic tool rather than the strength criterion itself. The method is positioned relative to established computer-aided STM work, particularly CAST and AStrutTie, to distinguish prior nodal stress-field and software capabilities from the specific integrated analytical-to-physical workflow implemented in STM Studio.

Keywords

strut-and-tie model; nodal zone; physical nodal geometry; face merging; AS 3600; CCC; CCT; CTT; non-hydrostatic node; stress tensor; principal stress; finite element; CST; stress field; concrete D-region; N-face node

1. Background and scope

Nodal zones in strut-and-tie modelling are two-dimensional regions through which several concentrated or distributed force paths are transferred. Classical hydrostatic nodal constructions are attractive because the loaded faces are perpendicular to their associated resultants and the face stresses can be made equal. General practical nodes, however, need not satisfy those geometric restrictions. Once a face is not perpendicular to its resultant, the boundary action necessarily contains both normal and shear traction. In an interactive design environment there is a second distinction that is equally important: the analytical STM is an equilibrium model, whereas the physical node is a finite concrete region that the engineer must construct, edit and eventually preserve as a design state.

The broader concept of two-dimensional nodal stress fields and computer-aided STM design is well established in the literature. Schlaich and Anagnostou (1990) developed stress fields for nodes with arbitrary numbers of struts, including piecewise homogeneous regions and transition fields. Tjhin and Kuchma developed CAST (Computer-Aided Strut-and-Tie) as an integrated graphical environment for defining D-regions, constructing and analysing truss models, selecting member dimensions and tie reinforcement, and producing design summaries. Their later nodal-zone capacity method estimated the stress distribution by dividing nodal zones into constant-stress triangles separated by lines of stress discontinuity and was implemented in CAST (Tjhin and Kuchma, 2007). Other work has used finite-element stress states to assess struts and nodes, including Yun and Ramirez (1996). Contemporary software such as AStrutTie also combines STM analysis/design with a plane-solid solver, principal stress-flow visualisation and evolutionary structural optimisation. Accordingly, the contribution documented here should not be characterised as the first use of stress fields, triangular elements, principal-stress visualisation or computer-aided STM design. The RKALC contribution is the particular integrated formulation and interactive workflow implemented in STM Studio: explicit separation of analytical actions from finite physical faces; fixed physical load/support contacts; designer editing and freezing; composite face merging; direct face traction resolution; whole-node tensor recovery; traction-mismatch diagnostics; a live centroid-fan CST stress-field heat-map reconstruction; and a separate nodal strength-check layer.

2. Definition of the general N-face node

Consider a planar physical nodal polygon Ω with N straight boundary faces Γi, i = 1,...,N, uniform out-of-plane thickness b, polygon area A, and centroid c = (xc,yc). The ith physical face has endpoints ai and bi, length wi, midpoint mi, unit tangent si and outward unit normal ni. The physical face count N need not equal the number M of analytical nodal actions. Where two adjacent analytical actions are assigned to one physical face, the face resultant Fi is the vector sum of those constituent actions. The traction, tensor and local-field formulations below use this physical-face resultant Fi. This distinction allows the analytical STM to remain unchanged while the engineer simplifies or regularises the finite nodal boundary.

$$A_i = bw_i \qquad (1)$$

$$\mathrm{t}_i = \frac{\mathrm{F}_i}{A_i} = \frac{\mathrm{F}_i}{bw_i} \qquad (2)$$

Equation (2) is the average traction vector associated with the resultant assigned to the physical face. Force units must be consistent with the geometry; in STM Studio, forces in kN are converted to N and lengths are in mm, giving stresses in MPa (N/mm^2). If several analytical actions share one physical face, their vector sum is used for this boundary traction while their original identities remain stored for analytical classification, offset-member mapping and stale-state checking.

2.1. Analytical actions and physical nodal faces

STM Studio treats the solved analytical truss and the finite nodal polygon as separate but linked objects. Member forces, applied loads and support reactions define the analytical nodal actions and remain the basis of equilibrium. The finite nodal polygon is a post-analysis construction owned by the designer. Editing the physical polygon therefore does not rotate or re-solve the analytical force vectors. Instead, the approved physical faces determine where those resultants enter the concrete region and how they are decomposed into normal and tangential traction.

A load or support face is an authoritative physical contact. Its midpoint, entered contact width and face angle are taken from the external action and its end vertices are locked in the node editor. If the force state, external contact width or external face angle later changes, a previously frozen node is retained exactly as saved but is marked stale for review rather than silently reshaped.

2.2. Interactive node construction, editing and freezing

The current interactive constructor begins from the solved nodal actions. The initial implementation supports three to six non-zero analytical actions and requires the analytical nodal forces to be in equilibrium. A uniform construction stress q and thickness b are used to generate a closed starting polygon with each initial face perpendicular to its associated force. Where an external physical face exists, q may be derived from that fixed contact and the generated edge is replaced by the exact entered load/support segment.

After initial construction, the polygon becomes designer-controlled geometry. Free member faces may be translated, resized or rotated graphically; a face can also be reset perpendicular to its analytical force or resultant. External faces and their end vertices remain fixed. The editor validates a finite non-zero polygon area independent of clockwise or counter-clockwise winding, checks self-intersection and convexity, checks containment within the concrete domain, and retains the analytical equilibrium residual as a separate diagnostic.

Freeze & Save is intentionally a geometry workflow action, not a declaration that the node has passed design. A geometrically valid node may therefore be frozen while its nodal strength status is PASS, FAIL or LIMIT REQUIRED. The latest design status is stored with the frozen geometry so that an iterative STM can preserve a useful physical construction, continue development elsewhere, and return later to revise a non-passing node.

2.3. Composite and merged physical faces

Adjacent analytical member faces may be merged into one physical face. The common polygon corner is removed and the new physical face carries the vector sum of the constituent analytical actions. The source member keys remain attached to the merged face so that analytical classification, member identity, reporting and offset-truss relationships are not lost.

A member face may also be merged into an adjacent physical load or support face. In this case the external contact is the governing geometry: its position, angle, width and endpoints remain exactly unchanged, and the member face is absorbed into that contact. The retained load/support face carries the combined resultant. The associated offset member terminates at the centre of the same retained physical face. External-to-external merging is not used because each external action represents a distinct physical contact.

Merged faces can be unmerged, restoring the removed member face and corner. The mapping of constituent source actions is also written into frozen geometry, so a saved/reopened model can rehydrate the physical face from the current analytical actions while still detecting whether the underlying force/contact state has become stale.

2.4. Current nodal strength-check layer

The nodal strength check is deliberately separated from the CST stress-field reconstruction. STM Studio classifies the node from the original analytical actions rather than from the reduced number of physical faces after merging. For a three-action node, the number of tension members gives the automatic categories CCC, CCT and CTT. TTT and nodes with more than three analytical actions are treated as manual-limit cases in the present implementation.

For the current AS 3600:2018 implementation, the automatic allowable nodal compressive stress is evaluated in STM Studio as $\sigma_{allow} = \phi_{st}\, \beta_n\, 0.9 f'_c$, with $\phi_{st} = 0.65$ and $\beta_n = 1.0$ for CCC, 0.8 for CCT and 0.6 for CTT. The governing principal-compression demand is compared with this allowable value. For TTT and complex N-face nodes, the designer enters the allowable nodal compressive stress manually. The resulting status is reported as PASS, FAIL or LIMIT REQUIRED and is stored with the frozen node, but it does not control whether valid physical geometry can be frozen.

This code-level check should be read independently from the average tensor, traction-mismatch percentages and local CST heat map described in the following sections. The latter are stress interpretation and diagnostic tools; the allowable-stress comparison is the design acceptance layer.

3. Face normal and shear stresses

Let θ_i be the acute angular deviation between the actual physical face normal and the line of action of the resultant F_i assigned to that face. Equivalently, the traction vector may be projected directly onto the face normal and tangent.

$$\sigma_{n,i} = \mathrm{t}_i \cdot \mathrm{n}_i \qquad (3)$$

$$\tau_i = \mathrm{t}_i \cdot \mathrm{s}_i \qquad (4)$$

$$\| \mathrm{t}_i \| = \sqrt{\sigma_{n,i}^2 + \tau_i^2} \qquad (5)$$

When only the magnitude F_i and deviation angle θ_i are used, the corresponding magnitudes are

$$\sigma_{n,i} = (F_i / b\, w_i) \cos \theta_i \qquad (6)$$

$$|\tau_i| = (F_i / b\, w_i) \sin \theta_i \qquad (7)$$

Thus $F/(bw)$ is not, in general, the normal stress. It is the magnitude of the average boundary traction. Only when $\theta_i = 0$ does the shear traction vanish and the traction become purely normal.

4. Whole-node average Cauchy stress tensor

A single uniform tensor cannot generally satisfy arbitrary tractions on more than a limited set of independent face orientations. Nevertheless, a rigorous volume-averaged stress tensor is useful as a global descriptor of the nodal state. Let $r_i = m_i - c$ be the vector from the polygon centroid to the centre of face i. For an equilibrated nodal body, the boundary-resultant form of the volume average is

$$\bar{\sigma} = \frac{1}{Ab} \sum_{i=1}^{N} (\mathrm{r}_i \otimes \mathrm{F}_i) \qquad (8)$$

For a continuum with negligible resultant couple, the Cauchy stress tensor is symmetric. STM Studio therefore retains the raw xy and yx terms as a diagnostic and uses the symmetric part for principal-stress calculations:

$$\bar{\sigma}_{sym} = \frac{1}{2}\left(\bar{\sigma} + \bar{\sigma}^{T}\right) \qquad (9)$$

$$\overline{\sigma}_{\mathrm{sym}} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \qquad (10)$$

$$\sigma_x = \frac{1}{Ab} \sum_{i=1}^{N} x_i \, F_{x,i} \qquad (11)$$

$$\sigma_y = \frac{1}{Ab} \sum_{i=1}^{N} y_i \, F_{y,i} \qquad (12)$$

$$\tau_{xy} = \frac{1}{2Ab} \sum_{i=1}^{N} \left(x_i F_{y,i} + y_i F_{x,i} \right) \qquad (13)$$

Here xi and yi are the components of ri. Translating the coordinate origin does not change the result when the boundary force system is in equilibrium.

5. Equilibrium checks

$$\sum_{i=1}^{N} F_{x,i} = 0 \qquad (14)$$

$$\sum_{i=1}^{N} F_{y,i} = 0 \qquad (15)$$

$$\sum_{i=1}^{N} \left(x_i F_{y,i} - y_i F_{x,i} \right) = 0 \quad (16)$$

The force and moment residuals are retained as numerical diagnostics. A significant residual indicates that the boundary system is not a self-equilibrated isolated nodal body, in which case any recovered stress field must be interpreted cautiously.

6. Principal stresses and Mohr-circle quantities

From the symmetric average tensor, define the mean normal stress and Mohr-circle radius:

$$\sigma_m = \frac{\sigma_x + \sigma_y}{2} \qquad (17)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \qquad (18)$$

$$\sigma_1 = \sigma_m + R \qquad (19)$$

$$\sigma_2 = \sigma_m - R \qquad (20)$$

$$\tau_{\max} = R \qquad (21)$$

$$\theta_p = \frac{1}{2} \mathrm{atan2}\left(2\tau_{xy}, \sigma_x - \sigma_y \right) \qquad (22)$$

STM Studio adopts tension-positive and compression-negative tensor signs. Accordingly, the most compressive principal stress is min(σ1,σ2), while the most tensile is max(σ1,σ2). Mohr's circle is a visual representation of these tensor invariants; it is not the solver used to establish the nodal field.

7. Quantifying departure from a uniform stress state

The average tensor can be tested against every actual boundary face. If the entire node were in a uniform stress state σ̄sym, the traction predicted on face i would be

$$\hat{\mathrm{t}}_i = \overline{\sigma}_{\mathrm{sym}} \mathrm{n}_i \qquad (23)$$

$$\Delta \mathrm{t}_i = \mathrm{t}_i - \hat{\mathrm{t}}_i \qquad (24)$$

$$\eta_i = \frac{\lVert \Delta \mathbf{t}_i \rVert}{\max(\lVert \mathbf{t}_i \rVert, t_{\mathrm{ref}})} \qquad (25)$$

where tref is a small reference traction used only to avoid unstable ratios on near-zero faces. STM Studio reports the maximum and root-mean-square values of ηi. These are compatibility diagnostics, not code-defined acceptance limits. A large mismatch means that the whole-node tensor remains a valid average descriptor but is not representative of the local boundary stress state; a spatially varying stress field is then required.

8. Local elastic stress-field reconstruction

For visualisation of the spatial variation, STM Studio generates a small continuum mesh by joining the polygon centroid to every boundary edge. An N-face polygon therefore produces N triangular elements. The present implementation uses linear constant-strain triangles (CST) under plane stress.

$$\varepsilon = \mathrm{B}\mathrm{u}_e \qquad (26)$$

$$\sigma = \mathrm{D}\varepsilon = \mathrm{D}\mathrm{B}\mathrm{u}_e \qquad (27)$$

For an isotropic plane-stress reconstruction,

$$\mathrm{D} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \qquad (28)$$

where E is an elastic modulus and ν is Poisson's ratio. The element stiffness is

$$\mathrm{k}_e = bA_e\mathrm{B}^{\mathrm{T}}\mathrm{D}\mathrm{B} \qquad (29)$$

and the assembled system is

$$\mathrm{K}\mathrm{u} = \mathrm{f} \qquad (30)$$

The boundary resultant on each straight face is treated as a uniform edge traction. For a two-node linear boundary edge, the consistent equivalent nodal loads are one half of the face resultant at each end:

$$\mathrm{f}_i^{\mathrm{edge}} = \frac{1}{2}\mathrm{F}_i, \qquad \mathrm{f}_j^{\mathrm{edge}} = \frac{1}{2}\mathrm{F}_i \qquad (31)$$

Because the isolated node is subjected principally to tractions, three rigid-body degrees of freedom must be removed to make the stiffness equations numerically solvable. STM Studio restrains the centroid translations and one boundary degree of freedom chosen to suppress rigid rotation. For a correctly equilibrated traction system these artificial restraints should carry negligible resultant reaction; the balancing reaction is therefore reported as a diagnostic.

9. Stress recovery and heat-map variables

Each CST element has a constant recovered stress vector

$$\sigma_e = [\sigma_x \quad \sigma_y \quad \tau_{xy}]^{\mathrm{T}} \qquad (32)$$

from which local principal stresses and maximum shear are evaluated using Equations (17)–(22). STM Studio can display the element field as a heat map of:

- principal compression, −min(σ1,σ2),
- principal tension, max(σ1,σ2),
- maximum in-plane shear, τmax,
- σx,
- σy, or
- τxy.

Principal compression is the preferred default for strut-and-tie interpretation because it reveals the magnitude and redistribution of compressive stress flow through the nodal region. Principal directions may be superimposed to show the orientation of the reconstructed stress trajectories.

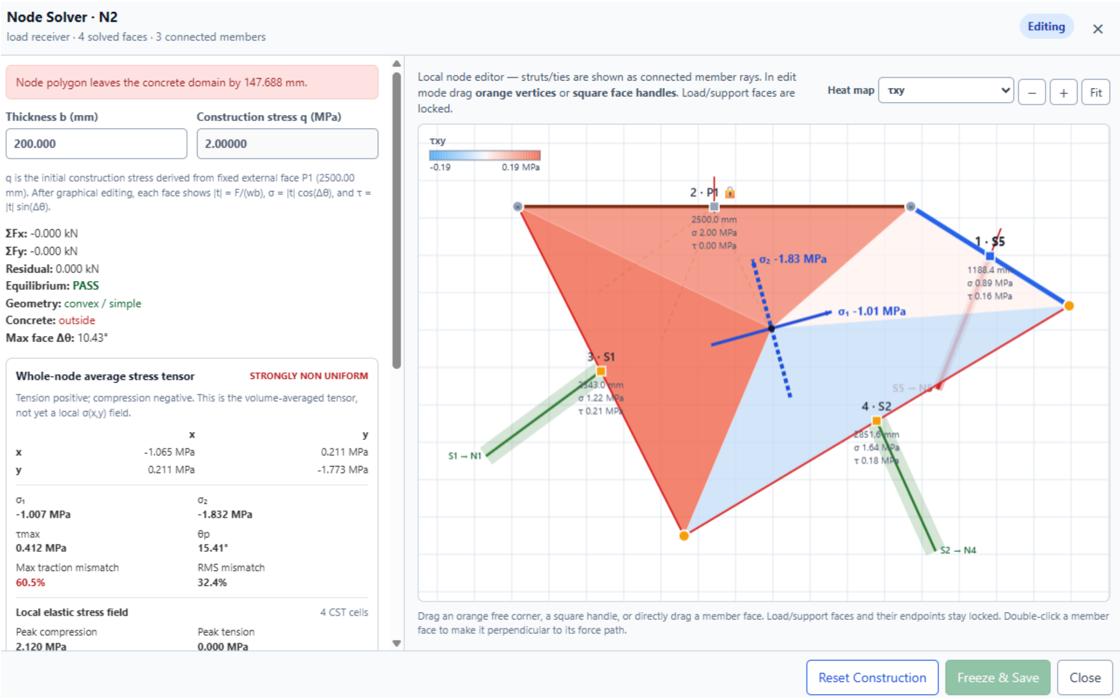

Figure 1. STM Studio general N-face node editor showing τxy heat map, boundary-face σ/τ values, average tensor and principal directions. The current workflow additionally supports composite physical-face merging and independent geometry freezing/design status as described in Section 2.

10. Interpretation of a strongly non-uniform node

A node may satisfy global force equilibrium while exhibiting a large traction mismatch relative to its average tensor. This is not a contradiction. It means that the stress field must vary spatially so that the actual tractions can be transmitted between differently oriented and differently stressed faces. The heat map provides a direct visual representation of that redistribution. Regions of elevated principal compression indicate concentrated compression transfer; regions of positive principal stress identify tensile demand in the elastic reconstruction; and high τ_{max} or τ_{xy} indicates significant stress rotation and shear transfer.

The local field should therefore be read together with, rather than instead of, the boundary-face tractions and the average tensor. The face results state what must enter or leave the node; the average tensor summarises the global stress state; the local reconstruction indicates one compatible elastic distribution through the interior.

11. Important limitations

- The local CST field is an elastic plane-stress reconstruction. It is not a nonlinear constitutive model for cracked or crushing concrete.
- The centroid-fan mesh is intentionally coarse. Element-wise stress jumps are expected because CST stresses are constant within each triangle.
- The reconstructed local field can depend on mesh topology, elastic assumptions and the rigid-body stabilisation used for the isolated traction problem.

- Local peak values from a coarse linear mesh must not be interpreted as mesh-objective ultimate concrete strengths.
- The traction-mismatch percentages are diagnostic measures devised for the STM Studio workflow; they are not acceptance limits from AS 3600, ACI 318 or another design standard.
- The current nodal allowable-stress check is a separate code-level layer. The CST heat-map extrema and traction-mismatch percentages are not themselves AS 3600 acceptance criteria. Strut strength, tie reinforcement, anchorage, confinement and other governing design requirements remain separate checks.
- A merged physical face is represented by the vector sum of its assigned analytical actions and by a single average traction over that face. The present local field does not separately resolve how multiple constituent actions distribute traction along the same physical contact.
- The current interactive constructor is limited to convex two-dimensional polygons and is initialised from three to six non-zero analytical nodal actions. Face merging may reduce the number of physical faces, but concave polygons, three-dimensional nodal zones and more general construction topologies require further development.
- Freeze & Save records a geometry state, not a design approval state. A frozen node may intentionally retain a FAIL or LIMIT REQUIRED status and must be reviewed before final design acceptance.
- Concave polygons, three-dimensional nodal zones, nonlinear cracking, tension softening, compression softening and reinforcement-concrete interaction require extensions beyond the present formulation.

12. Relationship to established nodal stress-field methods

The method belongs to the broader family of stress-field, continuum and computer-aided approaches to STM nodal zones. Schlaich and Anagnostou established that nodal regions may be treated as two-dimensional stress fields and presented constructive solutions for nodes with arbitrary numbers of struts. CAST, developed by Tjhin and Kuchma, is an important direct predecessor in computer-aided STM: published descriptions show a graphical environment for defining the D-region, constructing and analysing the internal truss, selecting strut dimensions and tie reinforcement, adjusting the model interactively and producing a design summary. In their 2007 nodal-zone method, Tjhin and Kuchma estimated nodal stresses using constant-stress triangles separated by stress-discontinuity lines and checked the resulting biaxial stress states against failure criteria. This prior work is conceptually close to the present use of triangular subregions, but its stated purpose is lower-bound nodal capacity assessment, whereas the present STM Studio centroid-fan CST field is formulated as an elastic equilibrium reconstruction and diagnostic visualisation.

AStrutTie provides a second useful benchmark at the software-system level. Published product descriptions indicate support for arbitrary D-regions, internally and externally determinate or indeterminate STM analysis, automated strut/tie/nodal checks, plane-stress and plane-strain analysis, principal-stress directions, evolutionary structural optimisation, DXF import, design templates, and ACI 318, AASHTO LRFD and Eurocode 2 provisions. Those capabilities overlap substantially with the broader objectives of STM Studio and demonstrate that topology/stress-flow guidance and integrated STM checking are established software concepts. STM Studio should therefore be distinguished by the architecture of its nodal workflow rather than by a broad claim of first computerisation or first stress visualisation: the analytical STM remains an explicit force model; fixed external contacts and free member faces are assembled into independently editable physical nodal polygons; adjacent analytical actions may share one physical face; member faces may be absorbed into load/support contacts without changing the external contact geometry; the engineer may freeze valid geometry independently of its current design status; and each retained physical face is then used for traction resolution, average-tensor recovery, compatibility diagnostics and the local CST heat map. The resulting chain from analytical force path to editable physical node to design check and local stress interpretation is the specific RKALC implementation documented here.

13. Position relative to CAST and AStrutTie

The comparison in this section is intended to establish prior art and clarify scope; it is not a claim that one program is universally superior to another. CAST is an academic predecessor closely related to the present nodal work, while AStrutTie is a useful benchmark for the breadth of a mature STM software package. The distinctions below are therefore framed in terms of documented method and workflow.

Aspect	CAST	AStrutTie	RKALC STM Studio
Primary role	Interactive graphical STM analysis/design environment; member dimensions, tie reinforcement and nodal-zone assessment.	Integrated STM analysis/design package with templates, code checks, solid analysis and optimisation tools.	Interactive STM environment coupling the analytical truss solution to designer-controlled finite nodal geometry, mergeable physical faces, a separate nodal strength-check layer and local stress interpretation.
Nodal stress treatment	Published 2007 method divides nodal zones into constant-stress triangles separated by stress discontinuities for lower-bound biaxial capacity assessment.	Published product information identifies automated nodal checks and a plane-solid solver with principal stresses/directions; detailed proprietary nodal formulation is not established by the sources reviewed here.	Physical-face resultants (including composite merged faces); normal/shear traction resolution; volume-averaged Cauchy tensor; traction-mismatch diagnostics; centroid-fan elastic CST reconstruction; heat maps and principal directions; separate nodal allowable-stress check.
Model interaction	Graphical D-region/truss construction and interactive adjustment with continuous recalculation.	Graphical STM construction with templates, DXF import and automated design functions.	Analytical STM remains separate from editable/frozen physical nodal polygons. Member faces can be merged with adjacent members or absorbed into fixed load/support contacts; freezing preserves valid geometry even when the design check is not passing.

Aspect	CAST	AStrutTie	RKALC STM Studio
Load-path guidance	Published CAST descriptions focus on designer-defined STM construction and refinement.	Principal stress flow and evolutionary structural optimisation are documented.	TopoKALC load-path guidance is used as an advisory precursor to engineer-defined STM construction.
Design standards	Historical CAST publications are tied principally to AASHTO/ACI-era STM development.	ACI 318, AASHTO LRFD and Eurocode 2 are documented.	AS 3600 is the present design focus. Current nodal checking automatically classifies CCC/CCT/CTT three-action nodes; TTT and complex N-face nodes use a designer-entered allowable stress. Additional standards are future work.
Position of present paper	Direct prior art for graphical STM and triangular nodal stress assessment.	Benchmark for integrated commercial STM functionality and stress-flow/topology assistance.	Documents the specific analytical-to-physical construction, face-merging, traction/tensor/CST workflow and separate nodal design-status layer; novelty claims still require formal literature review and validation.

Table 1. Method-level comparison based on published CAST literature, publicly available AStrutTie descriptions and the present STM Studio implementation. "Not established" means that the reviewed public source does not provide enough detail to support a stronger claim.

The comparison reinforces two publication principles. First, the triangular local field in STM Studio should be described as an implementation choice within an established family of nodal stress-field methods, not as the first triangular nodal formulation. Second, the strongest differentiator to test and document is the explicit analytical-to-physical workflow: analytical truss equilibrium is preserved while the engineer constructs finite nodal faces, may consolidate adjacent analytical actions into one physical face, preserves authoritative load/support contact geometry, freezes a valid geometric state independently of the current design result, and then derives boundary tractions, offset physical axes, code-level nodal status, tensor quantities and local field diagnostics from that construction.

14. Recommended verification Programme

1. Hydrostatic benchmark: equal normal compression on all compatible faces should give negligible shear and a near-uniform field.
2. Rotated-face benchmark: rotate one face while preserving its force vector and verify

$$\sigma n = |t|\cos\theta \text{ and } |\tau| = |t|\sin\theta.$$

3. Tensor benchmark: compare Equation (8) with direct numerical area averaging of a known constant stress field.
4. Mesh-refinement study: replace the centroid fan with successively refined triangular meshes and examine convergence of area-averaged and local stresses.

5. Patch test: impose tractions generated by a known constant tensor and confirm exact/near-exact recovery.
6. Literature benchmark: reproduce published non-hydrostatic nodal-zone examples and compare stress states and capacity checks.
7. Experimental validation: compare predicted critical stress regions and governing modes against tested D-regions and nodal failures.
8. CAST benchmark: reproduce at least one published Tjhin–Kuchma nodal-zone example and compare geometry, triangular stress states, equilibrium and governing biaxial demand. Differences should be explained in terms of lower-bound discontinuous stress fields versus the elastic CST reconstruction used by STM Studio.
9. Software benchmark: reproduce a common D-region in STM Studio and AStrutTie using matched geometry, material assumptions and design code where possible; compare truss forces, principal stress-flow trends, strut/tie demands and nodal conclusions without assuming identical internal formulations.
10. Face-merge equilibrium benchmark: merge two adjacent member faces and verify that the physical-face resultant equals the vector sum of the two original analytical actions, that the total nodal equilibrium is unchanged, and that unmerge restores the original action-to-face mapping.
11. External-face merge benchmark: merge an adjacent member into a load/support face and verify that the external face midpoint, angle, width and endpoints remain unchanged, while the member is mapped to the retained face and its offset physical axis terminates on that contact.
12. Nodal strength workflow benchmark: verify CCC, CCT and CTT automatic classification and allowable-stress calculation, verify TTT/complex manual-limit behaviour, and confirm that valid geometry can be frozen while the strength status is FAIL or LIMIT REQUIRED.

15. Proposed future extensions

The present formulation provides a foundation for a more advanced nodal-zone engine. Natural extensions include automatic mesh refinement, stress smoothing, nonlinear concrete constitutive laws, tension cut-off, compression softening, reinforcement layers, crack-direction tracking, lower-bound discontinuous stress fields, direct comparison of the biaxial stress state with a concrete failure envelope, and richer code-specific nodal formulations. For composite physical faces, a further extension would be to resolve multiple constituent action application points or non-uniform traction distributions along the retained face rather than representing only their combined resultant. These extensions should preserve the separation between analytical equilibrium, physical-node construction, diagnostic stress reconstruction and code-specific strength verification.

16. Conclusions

A general N-face nodal zone should not be reduced to a single bearing stress when its force resultants are not perpendicular to the actual physical faces. The boundary action on each face is a traction vector containing normal and shear components. Equally, the finite nodal geometry should not be conflated with the analytical truss itself. STM Studio preserves the solved analytical nodal actions while allowing the engineer to construct a separate physical polygon, retain exact load/support contacts, edit free member faces, consolidate adjacent analytical actions into a common physical face, and freeze a valid geometry state independently of the current design result. The physical face count may therefore differ from the number of underlying analytical actions, while equilibrium and action identity are preserved.

From the complete set of equilibrated physical-face resultants, a whole-node average Cauchy stress tensor can be recovered and expressed through principal stresses and directions. The degree to which that uniform tensor fails to reproduce the individual boundary tractions provides a useful measure of stress non-uniformity; where required, STM Studio supplements it with a centroid-fan CST elastic equilibrium reconstruction and interactive heat maps. In parallel, the current implementation performs a separate nodal allowable-stress check, automatically classifying CCC/CCT/CTT three-action nodes and requiring manual limits for TTT or complex cases. The local CST field remains diagnostic rather than a code strength criterion. The contribution should therefore be framed around this integrated chain from analytical action, to constructed/merged physical face, to nodal design status, to traction, tensor and local stress field, rather than around a claim of first use of stress-field or computer-aided STM concepts.

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10. RKALC Structural Engineering (2026). STM Studio implementation: general N-face nodal stress tensor and local elastic CST stress-field reconstruction. Software method under development.

Appendix A Computational algorithm

1. Solve the analytical STM and collect the non-zero member forces, applied loads and support reactions acting at the node; retain their original identities and check resultant force equilibrium.
2. Initialise the physical nodal polygon from the ordered analytical actions. The current interactive constructor supports three to six non-zero analytical actions before optional face merging.
3. Where a load/support face is present, replace the generated face with the exact entered physical contact segment and lock its position, width, angle and end vertices.
4. Allow graphical editing of free member faces. Validate finite non-zero polygon area independent of winding direction, self-intersection, convexity, concrete containment and analytical equilibrium.
5. Optionally merge two adjacent member faces, or merge an adjacent member into a load/support face. Preserve the constituent source-action keys and set the physical-face resultant to the vector sum of the assigned analytical actions. For member-to-external merging, retain the external contact geometry exactly.
6. Map each constituent offset member to the physical face on which it terminates. Where actions share a merged face, the current implementation uses the centre of that retained face as the common physical application point.
7. Freeze & Save any geometrically valid node independently of the current nodal strength result. Store the physical vertices, source mappings, analytical input signature and current design status; mark the frozen node stale if the analytical force/contact state subsequently changes.
8. Classify the node from the original analytical actions. For a three-action node, assign CCC, CCT or CTT from the number of tension members; treat TTT and more complex nodes as manual-limit cases.
9. For the current AS 3600:2018 implementation, evaluate $\sigma_{allow} = \phi_{st} \beta_n 0.9f'_c$ with $\phi_{st} = 0.65$ and $\beta_n = 1.0$, 0.8 or 0.6 for CCC, CCT or CTT respectively; otherwise read the designer-entered allowable stress. Report PASS, FAIL or LIMIT REQUIRED separately from geometry validity.
10. Read the ordered physical nodal polygon, physical-face resultants and thickness.
11. Calculate polygon area, centroid, each face midpoint, tangent and outward normal.
12. Convert each physical-face resultant to the average traction vector $t = F/(bw)$.
13. Resolve each traction into signed normal stress and signed shear stress.
14. Check resultant force and moment equilibrium of the isolated physical node.
15. Recover the whole-node average stress tensor from $\Sigma(r \otimes F)/(Ab)$, retain the raw nonsymmetric terms as diagnostics, and use the symmetric tensor for principal stresses.
16. For every physical face, calculate the traction predicted by the average tensor and compare it with the actual traction; report maximum and RMS mismatch.
17. Create one CST element between each physical polygon edge and the polygon centroid.
18. Assemble the plane-stress stiffness matrix using selected E and v.
19. Apply each physical-face resultant as equivalent half-end nodal loads on its boundary edge.
20. Remove the three rigid-body modes and solve $K u = f$.
21. Recover σ_x, σ_y and τ_{xy} in each CST element, then calculate σ_1, σ_2, τ_{max} and θ_p.
22. Render the selected field variable as a heat map and overlay principal directions.
23. Report balancing reactions, field extrema, traction-compatibility diagnostics and the separate nodal design status.

Appendix B Engineering interpretation of N-face plane-stress reconstruction

This appendix provides an engineering interpretation of the N-face formulation presented in Sections 3 to 9. Its purpose is to connect the general nodal-zone procedure to the familiar two-dimensional plane-stress problem. The formulation does not introduce an N-dimensional stress state. The continuum remains two-dimensional; the additional faces simply provide additional boundary-traction conditions that, in general, cannot all be represented by one spatially uniform stress tensor.

Key concept: an N-face node is still a two-dimensional plane-stress problem; the stress tensor generally varies with position inside the node.

B.1 Start from the physical nodal faces

Consider a physical nodal polygon with N straight faces. For each physical face i, the geometry provides a face length w_i, a unit outward normal n_i and a unit tangent s_i. The analytical STM provides the resultant force vector F_i assigned to that physical face. Where several analytical actions have been merged onto one physical face, F_i is the vector sum of those constituent actions, consistent with Section 2.3.

Before stress reconstruction, the complete boundary force system is checked for resultant force and moment equilibrium. If the boundary actions do not represent an equilibrated isolated nodal body, any recovered average or local stress field must be treated as a diagnostic rather than as a self-contained nodal solution.

B.2 Convert each face force to an average traction

For a node of out-of-plane thickness b, the average traction vector acting on face i is obtained by dividing the face resultant by the physical face area b w_i:

$$\mathrm{t}_i = \frac{\mathrm{F}_i}{bw_i} \tag{B.1}$$

The quantity $|F_i|/(b\ w_i)$ is therefore the magnitude of the average boundary traction. It is not, in general, the normal stress on the face. Normal stress is obtained only after the traction vector has been resolved relative to the actual physical face.

B.3 Resolve the traction into normal and shear components

The traction on each face is projected onto the face normal and tangent. This gives the signed normal and tangential components:

$$\sigma_{n,i} = \mathrm{t}_i \cdot \mathrm{n}_i \tag{B.2}$$

$$\tau_i = \mathrm{t}_i \cdot \mathrm{s}_i \tag{B.3}$$

Equivalently, if θ_i is the angle between the resultant force direction and the face normal, the magnitudes are $|\sigma_{n,i}| = |t_i| \cos \theta_i$ and $|\tau_i| = |t_i| \sin \theta_i$. A face that is perpendicular to its resultant has $\theta_i = 0$ and therefore carries purely normal traction. Rotating the physical face while retaining the same resultant introduces a shear-traction component.

B.4 Why one face does not define σx, σy and τxy

In a conventional plane-stress calculation the stress state at a point is described by three independent components: σx, σy and τxy. The traction acting on any plane through that point is then obtained from the Cauchy relation t = σ n.

$$t = \sigma n \tag{B.4}$$

A single nodal face supplies only the traction acting on one particular plane. It therefore gives σ_n and τ on that plane, but it does not by itself determine the complete Cartesian stress tensor. The full set of faces provides multiple boundary conditions on the nodal region.

For three or more arbitrarily oriented faces, those boundary tractions will generally be incompatible with one common uniform tensor. This is the central reason that a general N-face node cannot be reduced to a collection of independent bearing-stress checks or to one Mohr circle applied directly to every face.

B.5 Recover the whole-node average Cauchy stress tensor

Although a single uniform tensor will not normally reproduce every face traction exactly, the complete equilibrated boundary system defines a rigorous area-averaged Cauchy stress tensor. Let A be the polygon area, c its centroid, m_i the midpoint of face i and $r_i = m_i - c$. The whole-node average tensor is recovered from the boundary resultants as:

$$\overline{\sigma} = \frac{1}{Ab}\sum_{i=1}^{N}(r_i \otimes F_i) \tag{B.5}$$

This operation uses every physical face simultaneously. In Cartesian form, the result is the familiar two-dimensional stress tensor:

$$\overline{\sigma} = \begin{bmatrix} \overline{\sigma}_x & \overline{\tau}_{xy} \\ \overline{\tau}_{yx} & \overline{\sigma}_y \end{bmatrix} \tag{B.6}$$

For a classical Cauchy continuum with negligible resultant couple, the stress tensor is symmetric. STM Studio retains the raw xy and yx terms as an equilibrium diagnostic and uses their symmetric part for principal-stress calculations:

$$\overline{\tau} = \frac{\overline{\tau}_{xy} + \overline{\tau}_{yx}}{2} \tag{B.7}$$

At this point the N-face problem has returned to the ordinary plane-stress form familiar from elementary stress transformation.

B.6 Calculate principal stresses and the Mohr-circle quantities

Principal stresses are calculated from the symmetric average tensor in exactly the same manner as for any two-dimensional plane-stress state. Define the mean normal stress and Mohr-circle radius as:

$$\sigma_m = \frac{\overline{\sigma}_x + \overline{\sigma}_y}{2} \tag{B.8}$$

$$R = \sqrt{\left(\frac{\overline{\sigma}_x - \overline{\sigma}_y}{2}\right)^2 + \overline{\tau}^2} \tag{B.9}$$

$$\sigma_1 = \sigma_m + R, \qquad \sigma_2 = \sigma_m - R \tag{B.10}$$

$$\tau_{\max} = R \tag{B.11}$$

$$\theta_p = \frac{1}{2}\operatorname{atan2}\left(2\overline{\tau}, \overline{\sigma}_x - \overline{\sigma}_y\right) \tag{B.12}$$

STM Studio adopts tension-positive and compression-negative tensor signs. Accordingly, the most compressive principal stress is min(σ_1, σ_2) and the most tensile principal stress is max(σ_1, σ_2). The presence of five, six or ten nodal faces does not change the Mohr-circle mathematics: once a two-dimensional tensor has been established, the principal quantities are obtained by the standard plane-stress transformation.

B.7 Test whether the average tensor represents the actual faces

The average tensor is a global descriptor. The next step is therefore to ask whether that one tensor is also a satisfactory description of the actual boundary conditions. For each face, the traction predicted by the average tensor is:

$$\mathrm{t}_{\mathrm{pred},i} = \overline{\sigma}_{\mathrm{sym}}\mathrm{n}_i \tag{B.13}$$

This predicted traction is compared with the actual traction $t_i = F_i/(b\ w_i)$. If the two are close for all faces, the node is close to a uniform stress state. If the differences are large, the average tensor remains a valid area-average descriptor, but it cannot represent the local stress state throughout the node.

This distinction is important. Large traction mismatch is not a failure of equilibrium and it does not invalidate the average tensor. It indicates that the internal stress field must redistribute spatially in order to transmit the imposed tractions between faces having different orientations, widths and force directions.

B.8 Reconstruct the spatially varying stress field

Where a spatially varying field is required for interpretation, STM Studio joins the polygon centroid to each boundary edge. An N-face polygon therefore produces N triangular constant-strain elements (CSTs). The physical face resultant is applied to its boundary edge as a uniform edge traction, and the assembled plane-stress system is solved after removal of the three rigid-body modes.

Each triangular element then has its own constant stress vector:

$$\sigma_e = [\sigma_{x,e} \quad \sigma_{y,e} \quad \tau_{xy,e}]^T \tag{B.14}$$

For every element, the same ordinary plane-stress transformation is repeated to obtain σ_1, σ_2, τ_{max} and the principal direction. The collection of element results forms the heat map shown in STM Studio. In this sense, the N-face formulation does not require a new type of stress theory; it uses the standard two-dimensional stress tensor at several spatial locations.

B.9 Practical interpretation hierarchy

The results are most clearly interpreted in four levels. Each level answers a different engineering question and should not be substituted for another.

Level	Primary quantities	Engineering meaning
Boundary face	t_i, $\sigma_{n,i}$, τ_i	What traction must enter or leave the concrete through each actual physical face.
Whole node	$\sigma_{\bar{x}}$, $\sigma_{\bar{y}}$, $\bar{\tau}$, σ_1, σ_2	The area-averaged global stress state and its principal directions.
Compatibility	$\hat{t}_i$ versus t_i	Whether one uniform tensor is representative of the complete boundary traction system.
Local field	$\sigma_{x,e}$, $\sigma_{y,e}$, $\tau_{xy,e}$, $\sigma_{1,e}$, $\sigma_{2,e}$	One compatible elastic reconstruction of how stress may redistribute through the nodal region.

B.10 Conceptual five-face example

For a five-face node, the procedure may be read sequentially as follows. First, the five face resultants are checked for equilibrium. Second, each resultant is divided by its physical face area to obtain a traction vector and is resolved into normal and shear components. Third, all five resultants and their face-centre positions are used together to recover one average tensor. Fourth, that tensor is converted to principal stresses using the ordinary Mohr-circle relations. Fifth, the traction predicted by the average tensor is checked against each of the five actual face tractions. Finally, where the mismatch demonstrates significant non-uniformity, the five centroid triangles provide five local plane-stress states for the stress-field heat map.

N face resultants	**Face tractions**	**Average 2D tensor**	**Principal stresses**	**Traction compatibility**	**Local CST field**

The number of faces changes the number of boundary conditions and, in the centroid-fan reconstruction, the number of triangular elements. It does not change the dimensionality of the stress tensor.

B.11 Relationship to the nodal strength check

The stress interpretation described in this appendix remains separate from the code-level nodal strength check described in Section 2.4. The boundary traction components, average tensor, traction-mismatch diagnostic and local CST heat map explain the mechanics of the constructed nodal zone. They are not, by themselves, AS 3600 acceptance limits. The current design layer compares the governing principal-compression demand with the applicable nodal allowable stress for automatically classified CCC, CCT and CTT three-action nodes, while TTT and more complex cases require a designer-entered allowable stress.

B.12 Engineering summary

The general N-face procedure can therefore be reduced to a familiar sequence: resolve the actual traction on every physical face; use all equilibrated boundary resultants to obtain the whole-node average two-dimensional stress tensor; calculate the principal stresses by the standard plane-stress relations; test whether that average tensor reproduces the boundary tractions; and, where it does not, interpret the node using a spatially varying field composed of ordinary two-dimensional stress states.

In concise form: the complexity of an N-face node lies in the spatial variation of stress required to satisfy several boundary tractions, not in a higher-dimensional stress tensor.

Appendix C Worked numerical example - N13 four-face node

This appendix applies the formulation in Sections 3-7 and Appendix B to the four-face N13 node shown below. The objective is to reproduce, by hand, the boundary tractions, whole-node average Cauchy stress tensor, principal stresses and traction-compatibility measures reported by STM Studio. The example uses the engineering dimensions and face stresses annotated on the sketch; signs follow the STM Studio convention of tension positive and compression negative.

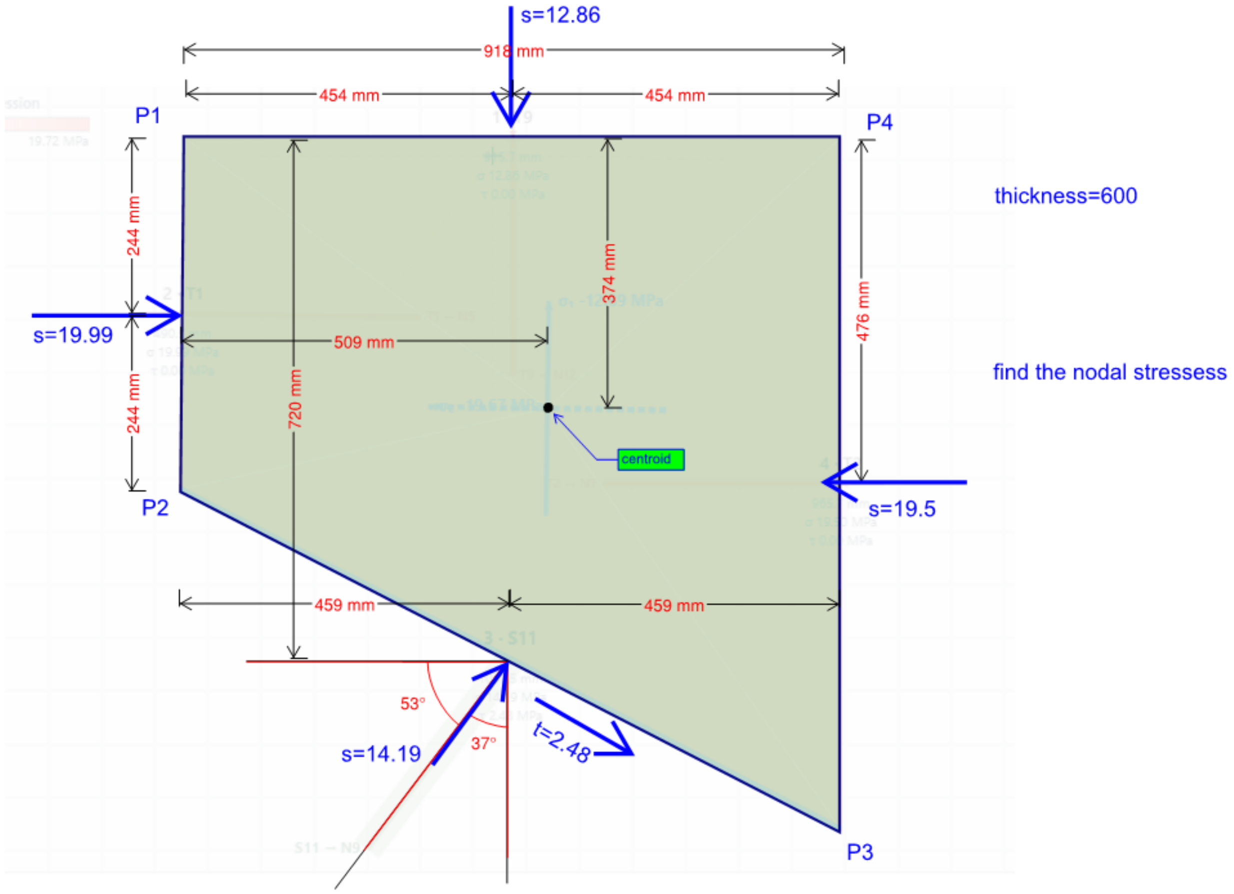


Figure C.1. N13 four-face nodal zone used for the resolved numerical example. Dimensions and stresses shown on the engineering sketch are rounded.

C.1 Input data and regularised calculation geometry

The hand annotation contains small mutually inconsistent rounded dimensions (for example, the upper width is shown as 454 + 454 mm whereas the lower horizontal projection is shown as 459 + 459 mm). A single consistent polygon is therefore adopted for the hand calculation. The regularisation retains the indicated 488 mm left face, 952 mm right face and 918 mm lower horizontal projection and reproduces the annotated centroid to within a few millimetres. This is sufficient for a transparent check of the calculation chain; STM Studio itself uses the full-precision saved vertices and force vectors.

$$\mathrm{P1} = (0, 0),\ \ \mathrm{P2} = (0, \text{-}488),\ \ \mathrm{P3} = (918, \text{-}952),\ \ \mathrm{P4} = (918, 0)\ \mathrm{mm} \quad \text{(C.1)}$$

The polygon is traversed counter-clockwise P1 → P2 → P3 → P4. The global x-axis is positive to the right and y is positive upward. For each face, s is the unit tangent in the polygon traversal direction and n is the outward unit normal.

$$A = 660\,960\ \mathrm{mm}^2, \quad c = (508.300, \text{-}372.459)\ \mathrm{mm}, \quad Ab = 396\,576\,000\ \mathrm{mm}^3 \quad \text{(C.2)}$$

Face	w (mm)	midpoint m (mm)	r = m - c (mm)	outward n
T1 - left	488.000	(0, -244)	(-508.300, +128.459)	(-1.000, 0.000)
S11 - inclined	1028.601	(459, -720)	(-49.300, -347.541)	(-0.45110, -0.89247)
T2 - right	952.000	(918, -476)	(+409.700, -103.541)	(+1.000, 0.000)
T9 - top	918.000	(459, 0)	(-49.300, +372.459)	(0.000, +1.000)

C.2 Boundary tractions and face resultants

The annotated values are treated as signed local face stresses. For the three orthogonal faces the tangential component is zero. For S11 the signed normal stress is -14.19 MPa and the tangential stress is +2.48 MPa in the P2 -> P3 tangent direction. Hence, for each face:

$$t_i = \sigma_{n,i}\, n_i + \tau_i\, s_i, \qquad F_i = t_i\, b\, w_i \quad (C.3)$$

For the inclined S11 face, **s = (0.89247, -0.45110)** and **n = (-0.45110, -0.89247)**. Therefore:

$$t(S11) = (-14.19)n + (2.43)s = (8.5698, 11.5680)\ \text{MPa} \quad (C.4)$$

$$|t(S11)| = \sqrt{(14.19^2 + 2.48^2)} = 14.405\ \text{MPa}, \quad \theta = \tan^{-1}(2.48/14.19) = 9.91° \quad (C.5)$$

Face	σ_n (MPa)	τ (MPa)	t = (t_x, t_y) MPa	F = (F_x, F_y) kN
T1	-19.99	0.00	(+19.990, 0.000)	(+5853.072, 0.000)
S11	-14.19	+2.48	(+8.570, +11.568)	(+5288.940, +7139.340)
T2	-19.50	0.00	(-19.500, 0.000)	(-11138.400, 0.000)
T9	-12.86	0.00	(0.000, -12.860)	(0.000, -7083.288)

C.3 Equilibrium residual from the rounded sketch values

Summing the four resultants obtained from the rounded sketch gives:

$$\Sigma F = (+3.612, +56.052)\ \text{kN} \quad (C.6)$$

$$\Sigma(r_i \times F_i) = -69.80\ \text{kN·m} \quad (C.7)$$

These residuals are small relative to the individual face resultants, which are of order 5 000-11 000 kN. They arise because the sketch dimensions and face stresses have been rounded independently. In the saved STM Studio model the full-precision analytical actions are checked for equilibrium before the tensor and local stress-field calculations are used.

C.4 Whole-node average Cauchy stress tensor

Using Equation (8), or Equation (B.5), each face contributes the outer product $r_i \otimes F_i$ divided by the nodal volume Ab. The four contributions, in MPa, are:

Face	σ_{xx}	σ_{xy}	σ_{yx}	σ_{yy}
T1	-7.5020	0.0000	+1.8959	0.0000
S11	-0.6575	-0.8875	-4.6350	-6.2566
T2	-11.5070	0.0000	+2.9081	0.0000
T9	0.0000	+0.8806	0.0000	-6.6525
Sum	-19.6665	-0.0070	+0.1690	-12.9091

$$\sigma(\text{raw}) = [[-19.6665, -0.0070], [+0.1690, -12.9091]] \text{ MPa} \quad (C.8)$$

The small difference between the raw xy and yx terms is another consequence of using the rounded sketch values. STM Studio retains these raw terms as a diagnostic and uses the symmetric part for principal-stress calculations:

$$\sigma(\text{sym}) = ½[\sigma(\text{raw}) + \sigma(\text{raw})^T] = [[-19.6665, 0.0810], [0.0810, -12.9091]] \text{ MPa} \quad (C.9)$$

C.5 Principal stresses and Mohr-circle quantities

From Equation (17)-(22):

$$\sigma_m = (-19.6665 - 12.9091)/2 = -16.2878 \text{ MPa} \quad (C.10)$$

$$R = \sqrt{\{[(-19.6665 + 12.9091)/2]^2 + 0.0810^2\}} = 3.3797 \text{ MPa} \quad (C.11)$$

$$\sigma_1 = -12.9082 \text{ MPa}, \quad \sigma_2 = -19.6675 \text{ MPa}, \quad \tau_{max} = 3.3797 \text{ MPa} \quad (C.12)$$

$$\theta_p = ½ \text{ atan2}[2(0.0810), -19.6665 + 12.9091] = 89.31° \quad (C.13)$$

The node is therefore strongly compression-dominated in both average principal directions, with very small average shear. The most compressive average principal stress is approximately 19.67 MPa for the rounded hand calculation.

C.6 Traction-compatibility check

For each face, the traction predicted by the symmetric average tensor is $\hat{t}_i = \sigma(\text{sym})n_i$. Comparing this with the actual boundary traction gives the following diagnostic mismatches:

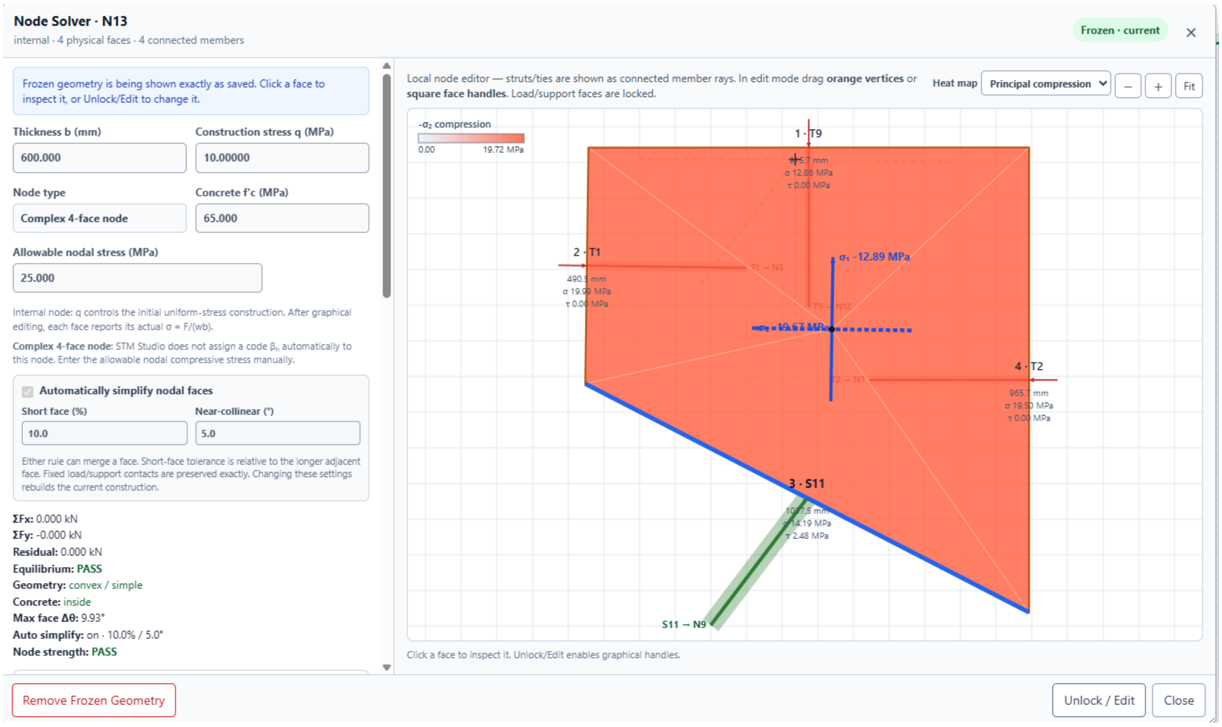
Face	actual t (MPa)	predicted $\hat{t}$ (MPa)	η_i
T1	(+19.990, 0.000)	(+19.667, -0.081)	1.67%
S11	(+8.570, +11.568)	(+8.799, +11.485)	1.70%
T2	(-19.500, 0.000)	(-19.667, +0.081)	0.95%
T9	(0.000, -12.860)	(+0.081, -12.909)	0.74%

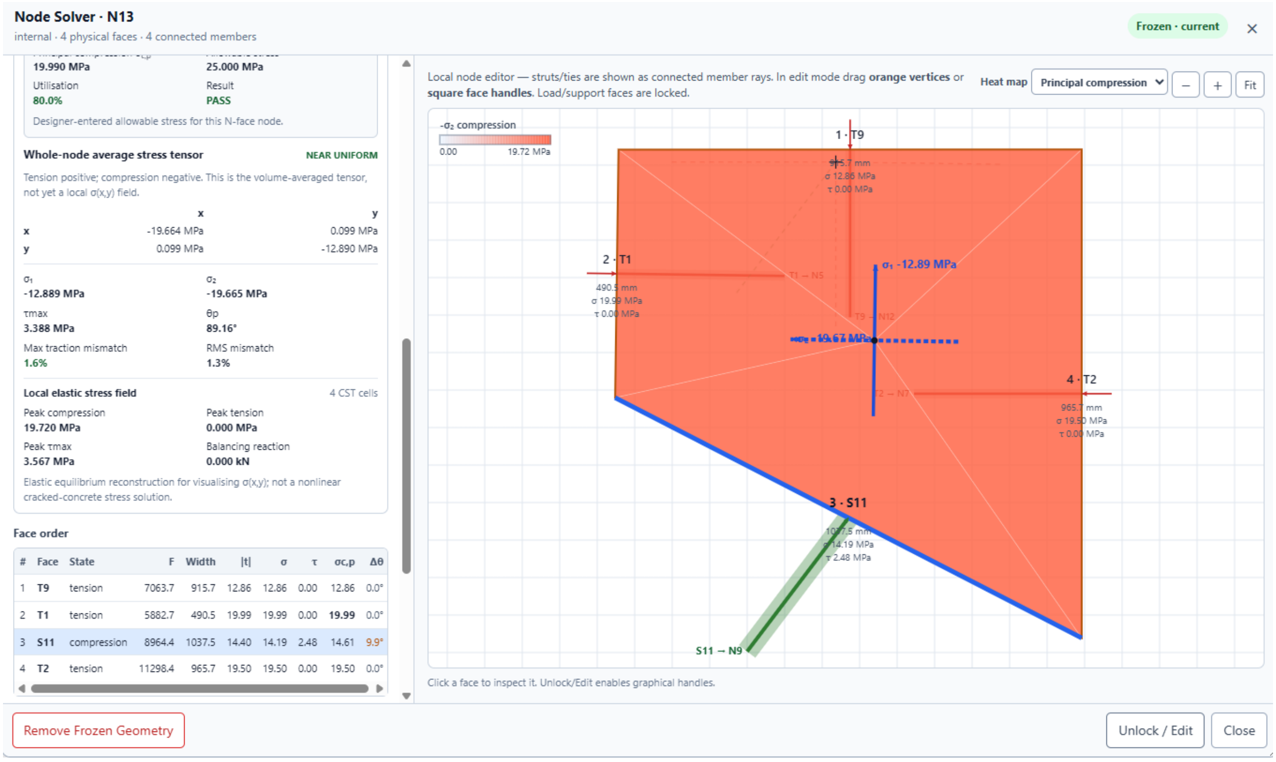
$$\eta_{max} = 1.70\%, \qquad \eta\text{RMS} = 1.33\% \quad (C.14)$$

C.7 Comparison with the full-precision STM Studio result

The corresponding full-precision STM Studio output for the same N13 node reports the symmetric average tensor and principal quantities shown below. The close agreement confirms that the hand calculation reproduces the implemented procedure; the remaining differences are consistent with the rounded dimensions and face stresses used in Figure C.1.

Quantity	hand calculation - rounded sketch	STM Studio - full precision
σ_x (MPa)	-19.6665	-19.664
σ_y (MPa)	-12.9091	-12.890
τ_{xy} (MPa)	+0.0810	+0.09
σ_1 (MPa)	-12.9082	-12.89
σ_2 (MPa)	-19.6675	-19.67
τ_{max} (MPa)	3.3797	3.388
θ_p (deg)	89.31	89.16
max traction mismatch	1.70%	1.6%
RMS traction mismatch	1.33%	1.3%




Node Solver · N13
internal · 4 physical faces · 4 connected members
Frozen · current
19.990 MPa
25.000 MPa
Utilisation
80.0%
Result
PASS
Designer-entered allowable stress for this N-face node.
Whole-node average stress tensor
NEAR UNIFORM
Tension positive; compression negative. This is the volume-averaged tensor, not yet a local σ(x,y) field.
x y
x -19.664 MPa 0.099 MPa
y 0.099 MPa -12.890 MPa
σ1 -12.889 MPa
σ2 -19.665 MPa
τmax 3.388 MPa
θp 89.16°
Max traction mismatch 1.6%
RMS mismatch 1.3%
Local elastic stress field
4 CST cells
Peak compression 19.720 MPa
Peak tension 0.000 MPa
Peak τmax 3.567 MPa
Balancing reaction 0.000 kN
Elastic equilibrium reconstruction for visualising σ(x,y); not a nonlinear cracked-concrete stress solution.
Face order
Face State F Width |t| σ τ σc,p Δθ
1 T9 tension 7063.7 915.7 12.86 12.86 0.00 12.86 0.0°
2 T1 tension 5882.7 490.5 19.99 19.99 0.00 19.99 0.0°
3 S11 compression 8964.4 1037.5 14.40 14.19 2.48 14.61 9.9°
4 T2 tension 11298.4 965.7 19.50 19.50 0.00 19.50 0.0°
Local node editor — struts/ties are shown as connected member rays. In edit mode drag orange vertices or square face handles. Load/support faces are locked.
Heat map
Principal compression
Fit
-σ2 compression
0.00
19.72 MPa
1 · T9
σ 12.86 MPa
τ 0.00 MPa
2 · T1
σ 19.99 MPa
τ 0.00 MPa
σ1 -12.89 MPa
4 · T2
965.7 mm
σ 19.50 MPa
τ 0.00 MPa
3 · S11
14.19 MPa
τ 2.48 MPa
S11 → N9
Click a face to inspect it. Unlock/Edit enables graphical handles.
Remove Frozen Geometry
Unlock / Edit
Close

C.8 Engineering interpretation

This resolved example demonstrates the full path from physical face stresses to the whole-node tensor. The face stresses are first converted to traction vectors in global coordinates; those tractions are integrated over each physical face to obtain force resultants; the spatially weighted outer products $r_i \otimes F_i$ are then summed and divided by the nodal volume. The resulting tensor is an area/volume average, not a statement that the stress is uniform everywhere in the node. The low traction mismatch for N13 shows that one uniform tensor is a good global representation of this particular constructed node. The local CST reconstruction remains the appropriate tool where the boundary tractions demand stronger spatial redistribution.

Verification scope. The worked example above independently verifies the boundary traction resolution, whole-node average tensor, principal-stress transformation and traction-compatibility calculations. The centroid-fan CST local field is not re-solved here because its element stresses depend on the exact saved vertices, elastic parameters and rigid-body stabilisation used by the software; those calculations remain covered by the verification programme in Section 14.

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