

FEAKalc3D

Modal Analysis Verification Report

Comparison with ETABS for two concrete building benchmarks

RK Software Solutions | 25 October 2026 | Revision 0

Verified by Rabee Kafina

1. Introduction

FEAKalc reproduces the principal modal response of the two ETABS benchmark models with close agreement. Fundamental periods differ by less than 1%, and the first six periods from twelve-vector Ritz analyses agree within approximately 2%. Twelve-vector horizontal participating mass exceeds 97% in both directions in both FEAKalc models and is within 0.7 percentage points of ETABS.

2. Verification findings

Measure	Model 1	Model 2
ETABS fundamental period	2.1120 s	1.3948 s
FEAKalc fundamental period	2.1026 s	1.3825 s
Fundamental period difference	-0.45%	-0.88%
Largest first six period difference	0.85%	2.00%
FEAKalc mass after 12 Ritz vectors	X 98.68% / Y 98.75%	X 98.02% / Y 97.41%
ETABS mass after 12 Ritz vectors	X 98.76% / Y 98.81%	X 98.47% / Y 98.08%

3. Evidence behind the comparison

The review combined production solver runs, exported stiffness and mass matrices, an independent sparse eigen solution, mass-source reconciliation, mesh and support inspection, and numerical regression checks. Code scrutiny covered shell shear stiffness, column to slab coupling and load-dependent Ritz generation. The final Ritz correction was tested on both full benchmarks without changing their physical stiffness or mass.

The supplied plan, three-dimensional, mesh and first-three-mode views are presented alongside numerical results. The findings establish comparable linear elastic modal behaviour for these two configurations. They do not imply identical meshes or uniform agreement of every higher Ritz mode.

4. Analysis basis and comparison method

Both benchmarks use concrete shell and frame elements with horizontal UX and UY starting acceleration loads. The primary numerical comparison uses twelve Ritz vectors in each application. Model 1 also has a separately generated six-vector comparison. That run is treated separately because changing a Ritz basis size can change its higher modes and mass distribution [R1, R2].

Input or convention	Verification basis
Elastic modulus	32,800 MPa in the concrete frame and shell stiffness
Shell properties	200 mm thickness; Poisson ratio 0.20; membrane and bending modifiers 1.0

Input or convention	Verification basis
Column section	600 by 600 mm concrete columns
Concrete unit weight	25 kN/m ³ for mass generation
Mass source	Self weight plus Dead and 0.30 Live contributions
FEAKalc mesh	1.25 m Smooth Quad slabs and rectangular walls
Analysis	Three-dimensional linear elastic model; UX/UY Ritz excitations

FEAKalc resolves shell modulus from the assigned 40 MPa concrete grade, giving 32,800 MPa. The older global fallback modulus field in Model 2 is therefore not the modulus used by its concrete shells. ETABS Model 2 includes small 50 by 50 mm meshing beams; the mass reconciliation below includes the remaining model differences.

5. Quantities compared

$$K\phi = \lambda M\phi \quad \lambda = \omega^2 \quad T = 2\pi / \omega$$

$$Effective\ mass\ ratio = \frac{(\phi^T M r)^2}{(\phi^T M \phi)(r^T M r)}$$

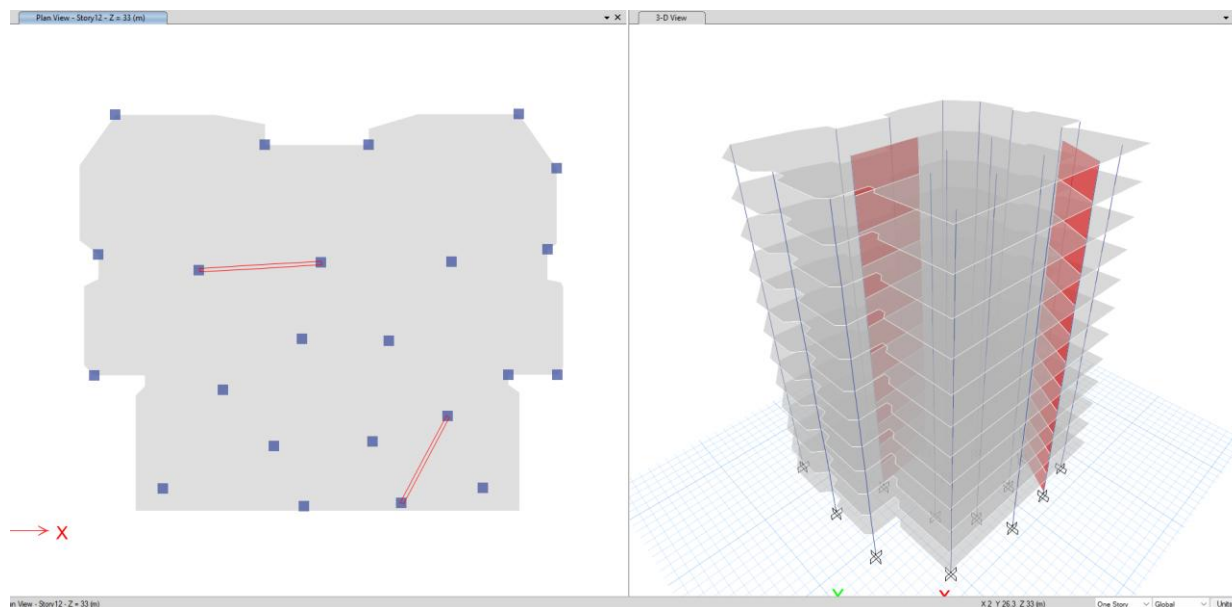
K and M are the assembled stiffness and mass matrices; ϕ is a mode shape and r selects the excitation direction. Effective mass ratios use the free translational mass in that direction. Reported percentage period difference is 100 times (FEAKalc period divided by ETABS period minus one). Mass-capture differences are percentage points.

Total assembled horizontal mass is compared separately, including restrained-node mass. Close totals do not establish identical mass placement or complete mass matrices. FEAKalc retains vertical translational mass in Model 2, while its ETABS source excludes vertical mass; this report therefore verifies horizontal excitation response.

6. Model 1 geometry and structural arrangement

Model 1 is an irregular concrete floor plate supported by columns and a limited wall system. It contains twelve slab levels at elevations 0 to 33 m above the base at -3 m, with 3 m vertical spacing. The retained source contains 276 frame objects, 12 slab polygons and 24 wall panels.

ETABS



FEAKalc

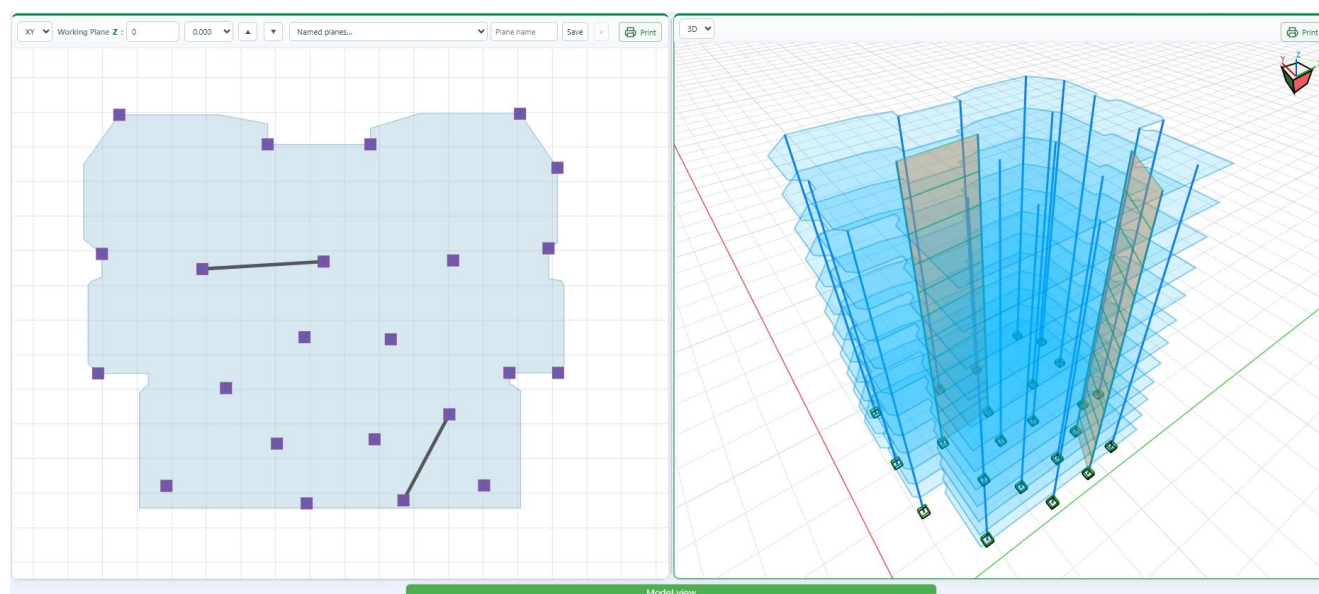
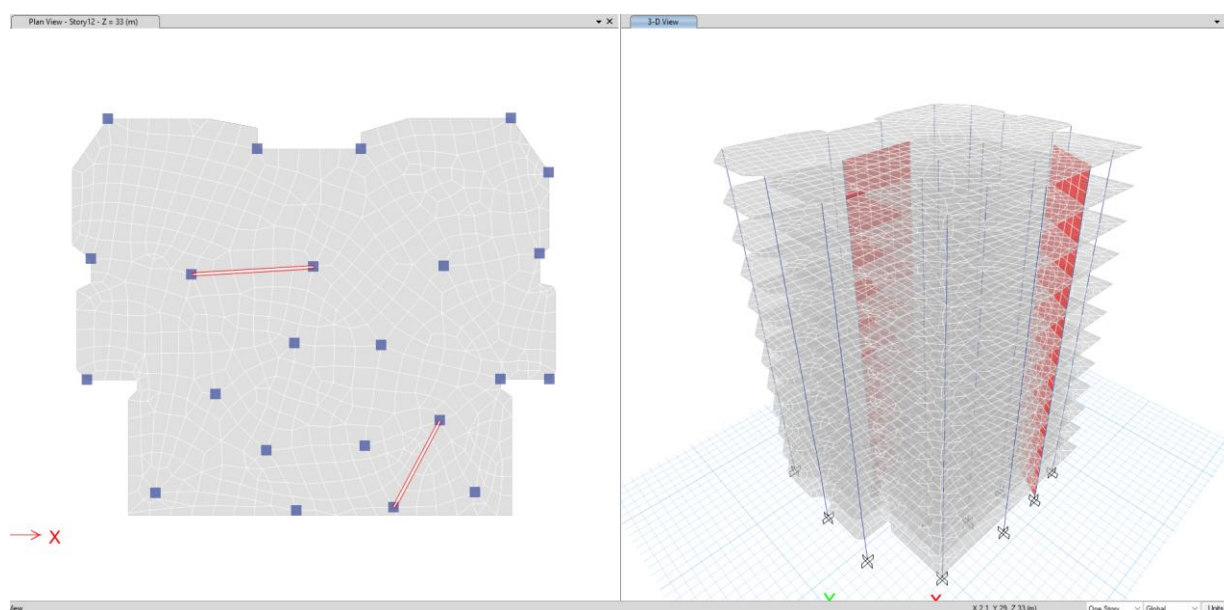


Figure 1. Model 1 plan and three-dimensional views. ETABS is above and FEAKalc below. The same irregular outline, column arrangement and wall locations are represented.

6.1. Model 1 analysis mesh and connectivity

The verified FEAKalc model contains 6,455 nodes, 6,444 shell elements and 372 generated frame elements, with 38,556 free degrees of freedom. Slab and wall meshes are checked at their interfaces; a nominal mesh size does not make the applications use identical element layouts.

ETABS



FEAKalc

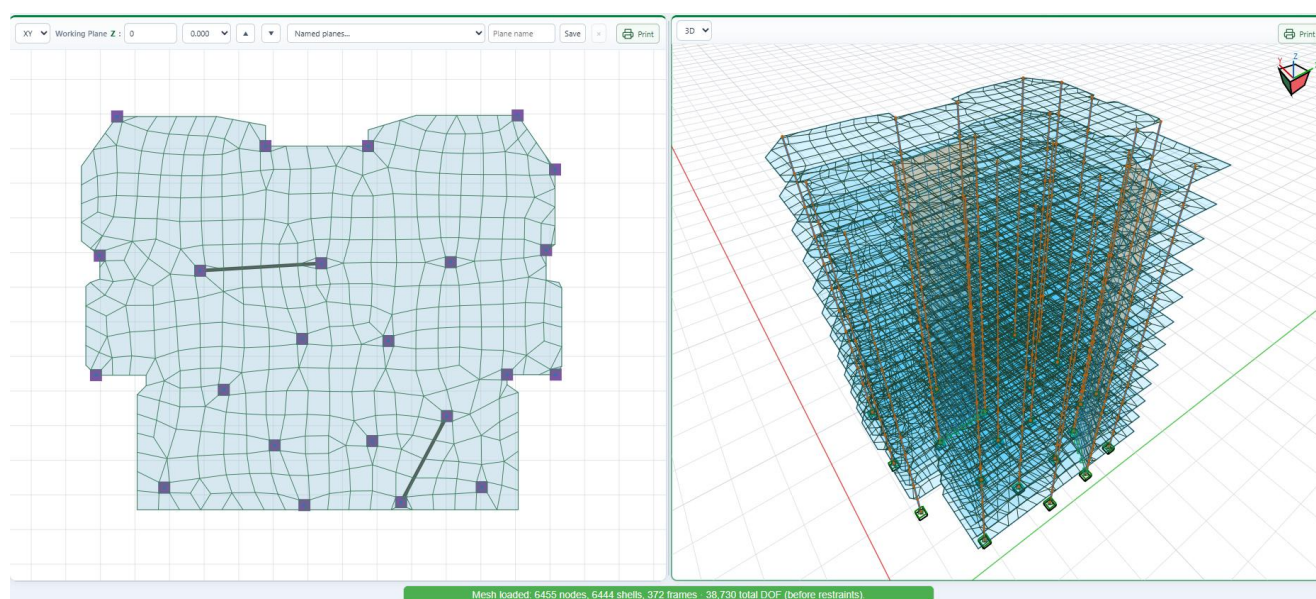


Figure 2. Model 1 analysis meshes. FEAKalc uses the 1.25 m Smooth Quad slab mesh and rectangular walls. Differences in element topology are visible and retained in the comparison.

6.2. Model 1 first and second modes

The first two modes show the corresponding coupled lateral response of the irregular floor plate. Periods below are from the twelve-vector runs. Screen deformation amplitude and contour colour are not directly comparable between applications.

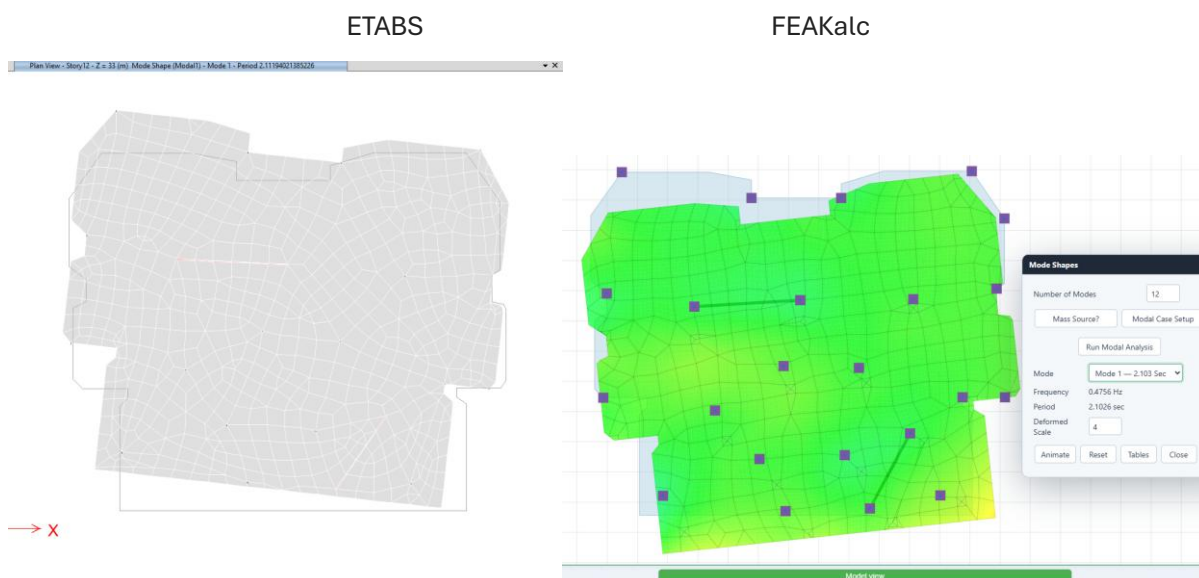


Figure 3. Model 1, mode 1: ETABS 2.1120 s; FEAkalc 2.1026 s. Display scale, phase and contour conventions are independent.

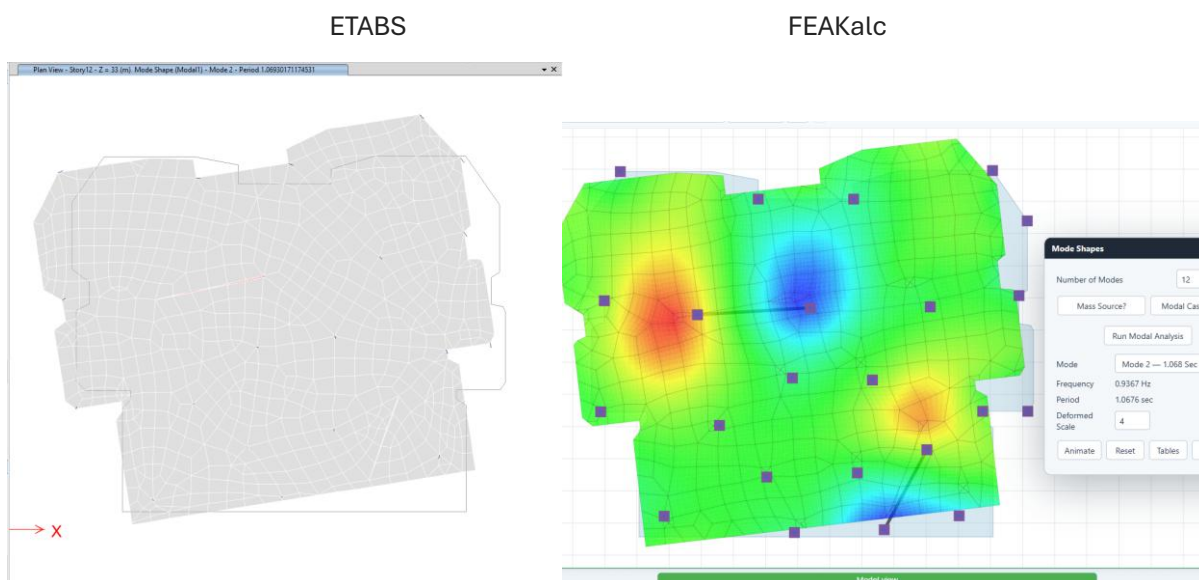


Figure 4. Model 1, mode 2: ETABS 1.0690 s; FEAkalc 1.0676 s. Display scale, phase and contour conventions are independent.

6.3. Model 1 third mode and modal periods

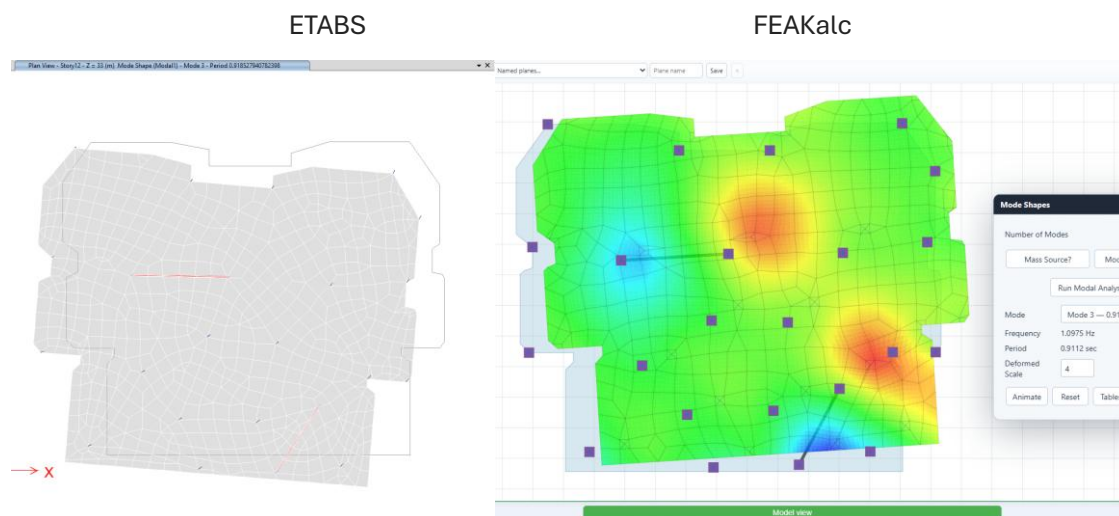


Figure 5. Model 1, mode 3: ETABS 0.9190 s; FEAkalc 0.9112 s. Display scale, phase and contour conventions are independent.

Mode	ETABS s	FEAkalc s	Difference
1	2.112	2.102558	-0.45%
2	1.069	1.067581	-0.13%
3	0.919	0.911170	-0.85%
4	0.647	0.643567	-0.53%
5	0.339	0.337374	-0.48%
6	0.243	0.242502	-0.21%
7	0.209	0.207299	-0.81%
8	0.203	0.202620	-0.19%
9	0.128	0.126776	-0.96%
10	0.103	0.103170	+0.16%
11	0.074	0.072837	-1.57%
12	0.061	0.060438	-0.92%

The first six periods differ by at most 0.85% against the ETABS values reported to three decimal places. All twelve periods remain close, with the larger relative differences in the shortest reported periods also affected by ETABS display rounding.

6.4. Model 1 horizontal mass participation

The supplied twelve-vector output tables independently show almost the same cumulative horizontal mass capture. These values are actual effective mass participation, rather than static load participation or modal direction factors.

ETABS

Modal Participating Mass Ratios															
Case	Mode	Period sec	UX	UY	UZ	SumUX	SumUY	SumUZ	RX	RY	RZ	SumRX	SumRY	SumRZ	
Modal1	1	2.112	0.0303	0.4508	2.373E-06	0.0303	0.4508	2.373E-06	0.1035	0.0057	0.2871	0.1035	0.0057	0.2871	
Modal1	2	1.089	0.2014	0.1263	3.884E-05	0.2318	0.5772	4.121E-05	0.0443	0.0671	0.3465	0.1478	0.0729	0.6336	
Modal1	3	0.919	0.4492	0.1387	1.163E-06	0.681	0.7158	4.238E-05	0.0679	0.1489	0.0803	0.2157	0.2218	0.7139	
Modal1	4	0.647	0.0009	0.0708	7.75E-06	0.6819	0.7866	0.0001	0.1759	0.0208	0.0349	0.3916	0.2426	0.7488	
Modal1	5	0.339	0.0033	0.0244	7.179E-06	0.6852	0.811	0.0001	0.0298	0.0026	0.0171	0.4214	0.2452	0.7659	
Modal1	6	0.243	0.0686	0.0298	0.0002	0.7537	0.8408	0.0003	0.0373	0.0826	0.0944	0.4587	0.3278	0.8603	
Modal1	7	0.209	0.0005	0.0259	5.435E-06	0.7542	0.8667	0.0003	0.0497	0.0001	0.005	0.5084	0.328	0.8654	
Modal1	8	0.203	0.1185	0.043	6.557E-07	0.8727	0.9097	0.0003	0.046	0.1456	0.0287	0.5544	0.4735	0.8941	
Modal1	9	0.128	0.0044	0.0228	3.769E-05	0.877	0.9325	0.0003	0.037	0.0095	4.624E-06	0.5914	0.4831	0.8941	
Modal1	10	0.103	0.0493	0.0006	0.0002	0.9263	0.933	0.0005	0.0009	0.045	0.008	0.5923	0.5281	0.9021	
Modal1	11	0.074	0.0008	0.0538	0.0001	0.9271	0.9868	0.0006	0.1018	0.0012	0.0252	0.694	0.5292	0.9272	
Modal1	12	0.061	0.0605	0.0012	0.0007	0.9878	0.9881	0.0013	0.0015	0.1552	0.0012	0.6955	0.6845	0.9285	

FEAKalc

Output Tables

Table

Modal information — Modal Participating Mass Ratios

Filter

Search rows

Export CSV

Print

Choose tables

Ratios are fractions (0–1), with cumulative sums in mode order. Translation uses free translational mass; rotation uses mass moment of inertia. Rotations are about the Center of free translational mass (19.126854, 12.430025, 16.299243) m. Export and print include all filtered rows, across pages.

Mode	Period s	UX	UY	UZ	RX	RY	RZ	Sum UX	Sum UY	Sum UZ	Sum RX
1	2.1025585	0.02861803	0.45394389	0.0000044930886	0.099601917	0.005042561	0.2847235	0.02861803	0.45394389	0.0000044930886	0.099601917
2	1.067581	0.21154636	0.12212484	0.0000362522	0.040717494	0.067873978	0.34040023	0.2401644	0.57606874	0.000040745289	0.14031941
3	0.91116963	0.4409752	0.13736899	0.0000022753799	0.068620631	0.1386365	0.090035387	0.6811396	0.71343773	0.000043020669	0.20894004
4	0.64356671	0.00041693954	0.073359888	0.000012580455	0.17648637	0.021902864	0.03339214	0.68155654	0.78679762	0.000055601124	0.38542641
5	0.33737356	0.0034542775	0.024133591	0.000012616082	0.030123698	0.0027234602	0.017108248	0.68501082	0.81093121	0.000068217206	0.41555011
6	0.24250179	0.069286328	0.029511359	0.00017884685	0.037691305	0.084868691	0.094006569	0.75429715	0.84044257	0.0002706406	0.45324141
7	0.20729852	0.0016677914	0.029461307	0.000028567084	0.056210229	0.00099567159	0.00374949	0.75596494	0.86990387	0.00027563114	0.50945164
8	0.20261959	0.11644139	0.039379322	4.2676563e-7	0.041925912	0.14747233	0.030282404	0.87240633	0.90928319	0.0002760579	0.55137755
9	0.1267755	0.0046285569	0.023034793	0.000057134572	0.038114383	0.010122429	0.0000060370493	0.87703488	0.93231799	0.00033319248	0.58949194
10	0.10316973	0.048840666	0.00071484313	0.00014320008	0.0010591455	0.04535001	0.0078816078	0.92587555	0.93303283	0.00047639255	0.59055108
11	0.072836661	0.00082321999	0.053326272	0.000087627712	0.10299397	0.0013918781	0.025019243	0.92669877	0.9863591	0.00056402026	0.69354505
12	0.060437866	0.060116451	0.0011053103	0.00052792101	0.0013756471	0.15636184	0.0012630249	0.98681522	0.98746441	0.0010919413	0.6949207

Previous1–12 of 12 rowsNext

Figure 6. Model 1 twelve-vector participating-mass tables. Ratios in the screenshots are fractions; the summary below expresses them as percentages.

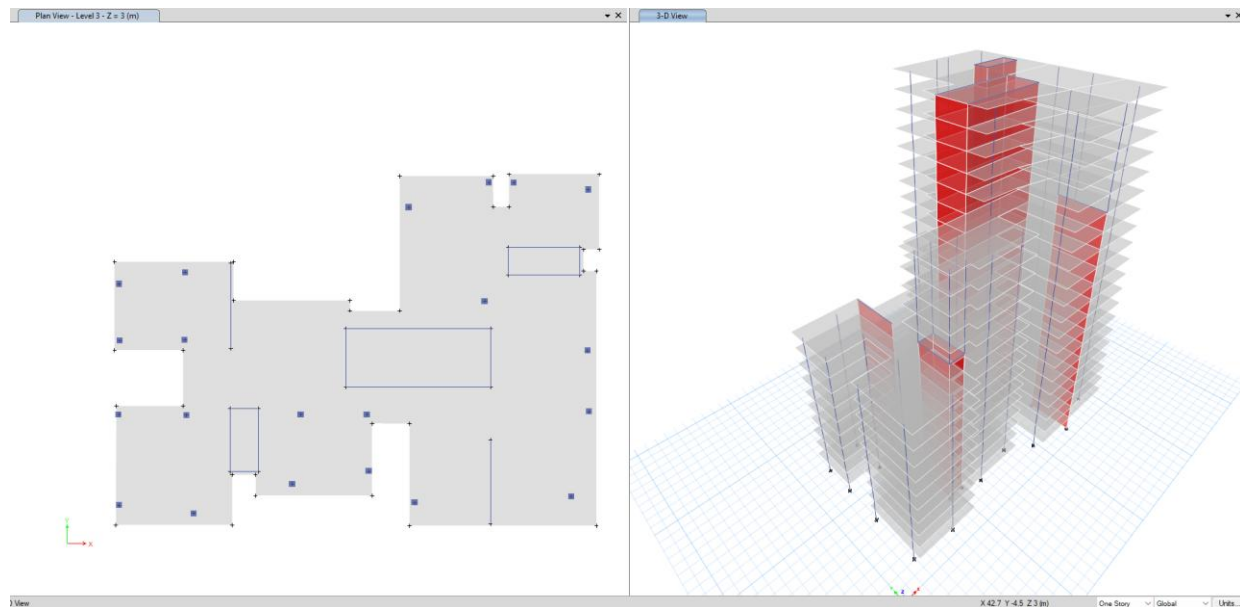
Ritz vectors	ETABS X / Y	FEAKalc X / Y
6 generated separately	90.52% / 92.36%	90.43% / 92.44%
12 generated separately	98.76% / 98.81%	98.68% / 98.75%

The six-vector result agrees within 0.09 percentage points in each direction. Taking only the first six modes of a twelve-vector run would be a different comparison: ETABS then has 75.37% X and 84.08% Y, while FEAKalc has 75.43% X and 84.04% Y.

7. Model 2 geometry and structural arrangement

Model 2 has a stepped floor arrangement with a more extensive concrete wall system. Its twenty slab levels extend from 3 to 60 m above a base at 0 m. The retained FEAKalc source contains 355 frame objects, 20 slab polygons and 225 wall panels.

ETABS



FEAKalc

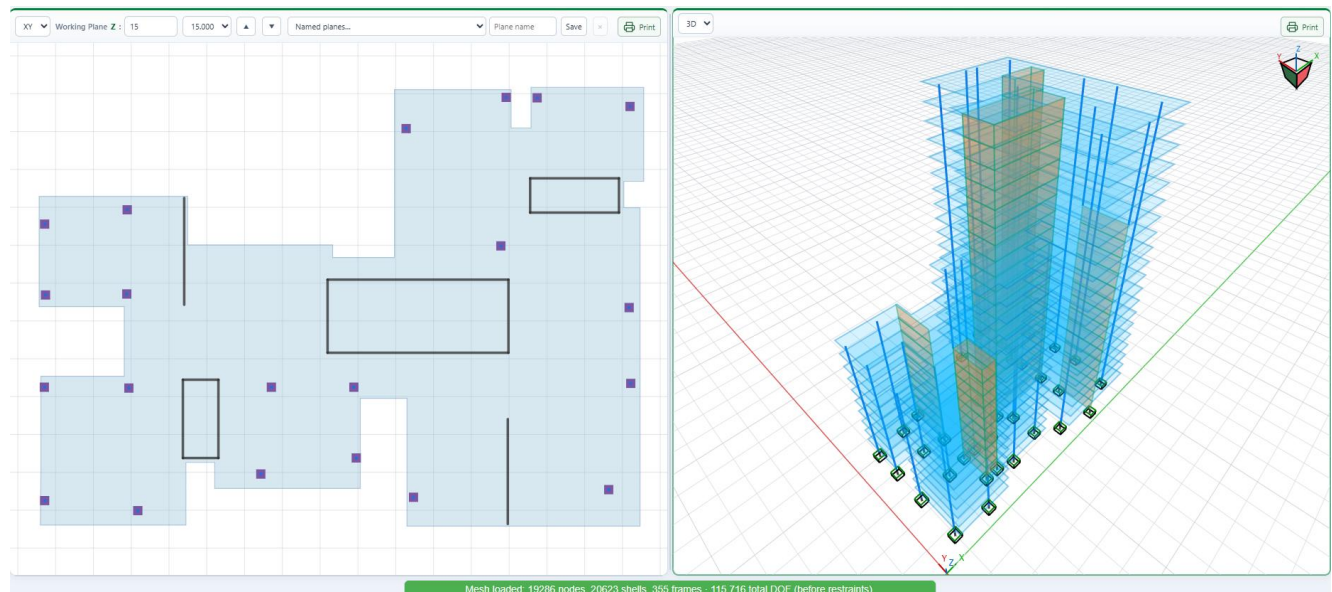
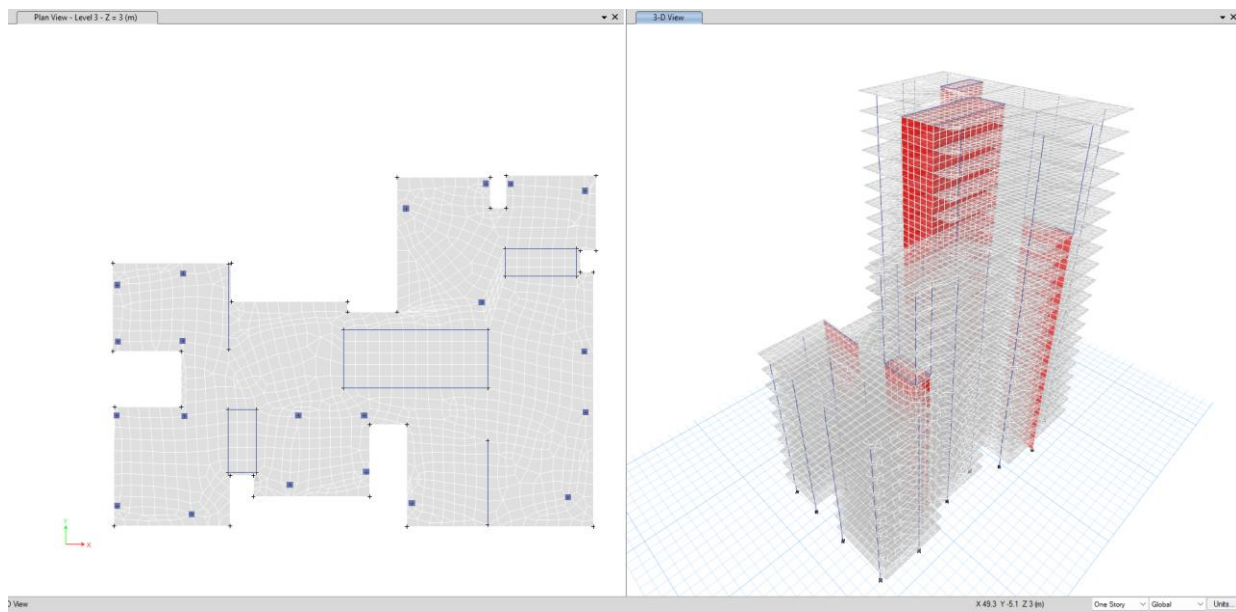


Figure 7. Model 2 plan and three-dimensional views. ETABS is above and FEAKalc below. The stepped building configuration and the increased extent of the wall system are common to both models.

7.1. Model 2 analysis mesh and connectivity

The verified FEAKalc model contains 19,286 nodes, 20,623 shell elements and 355 frame elements, with 115,266 free degrees of freedom. Both ETABS slabs and walls are meshed in the retained reference analysis. FEAKalc uses the same 1.25 m target size as Model 1.

ETABS



FEAKalc

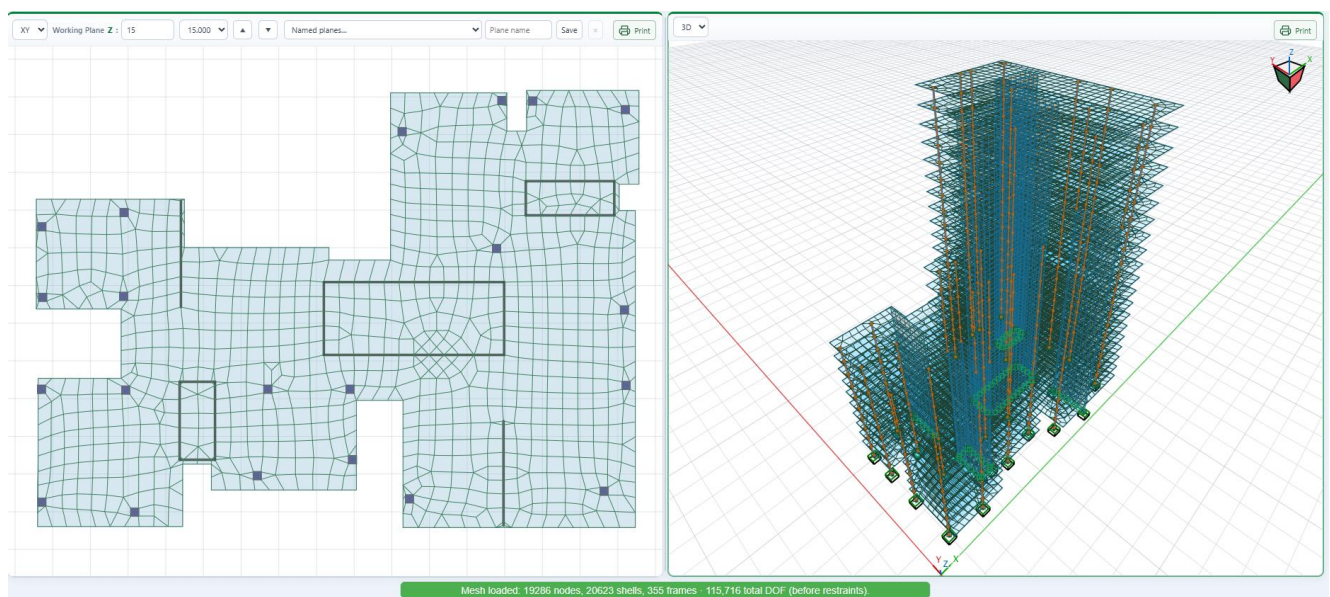


Figure 8. Model 2 analysis meshes. Rectangular wall discretisation and slab interface connectivity are included in the verified configuration. The actual meshes remain different between applications.

7.2. Model 2 first and second modes

The first mode is predominantly Y translation and the second predominantly X translation in the ETABS participation table. The corresponding periods and graphical patterns are closely reproduced by FEAKalc. Independent display magnification is retained.

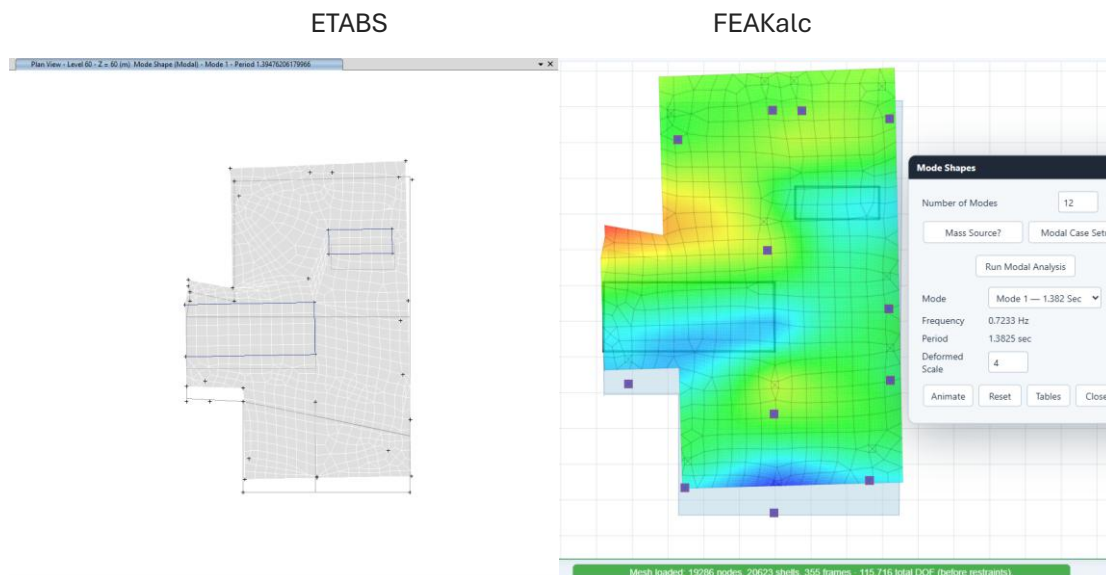


Figure 9. Model 2, mode 1: ETABS 1.3948 s; FEAKalc 1.3825 s. Display scale, phase and contour conventions are independent.

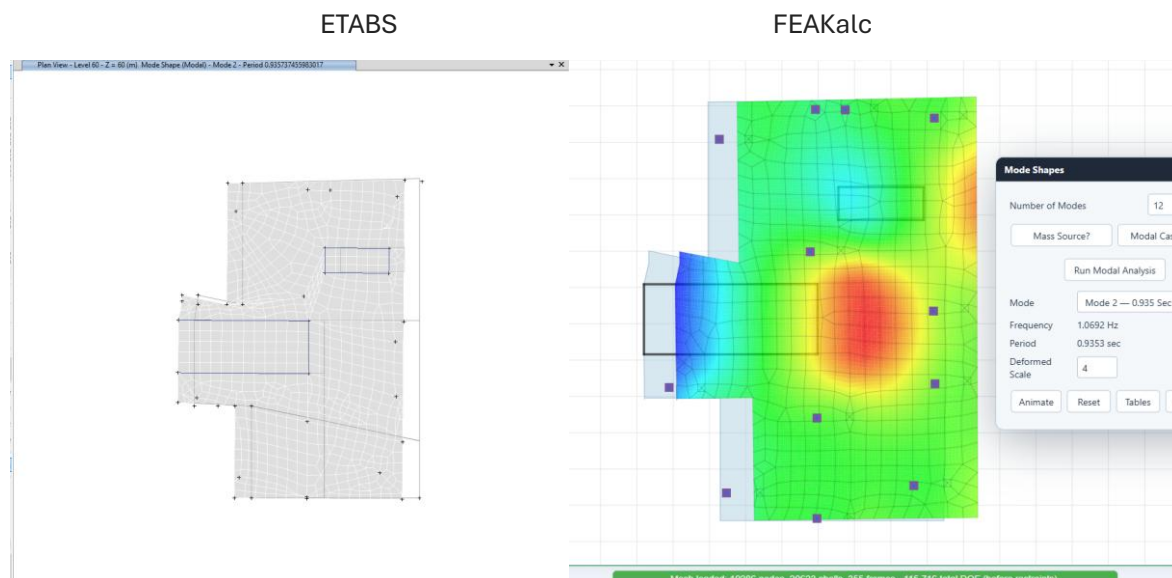


Figure 10. Model 2, mode 2: ETABS 0.9357 s; FEAKalc 0.9353 s. Display scale, phase and contour conventions are independent.

7.3. Model 2 third mode and modal periods

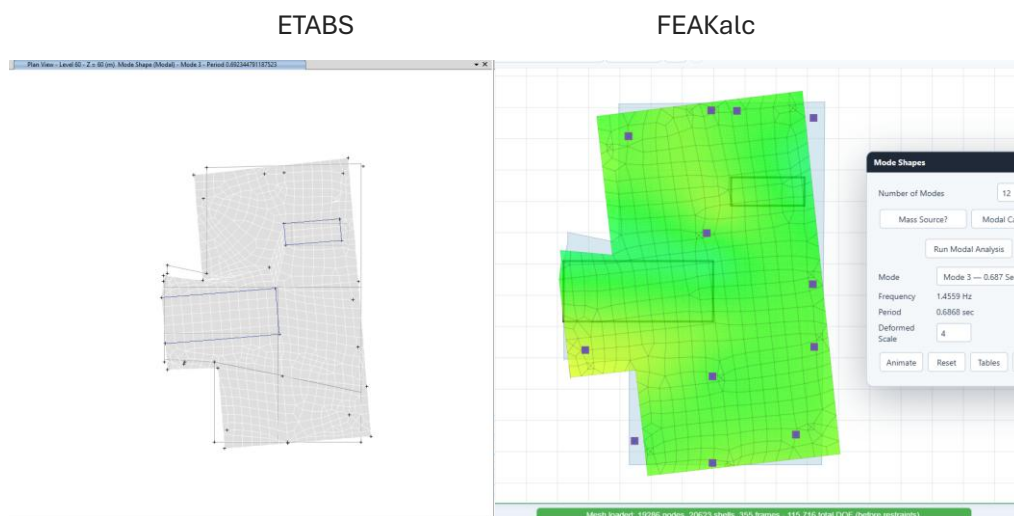


Figure 11. Model 2, mode 3: ETABS 0.6923 s; FEAkalc 0.6868 s. Display scale, phase and contour conventions are independent.

Mode	ETABS s	FEAkalc s	Difference
1	1.394762	1.382459	-0.88%
2	0.935737	0.935272	-0.05%
3	0.692345	0.686845	-0.79%
4	0.354086	0.354140	+0.02%
5	0.297363	0.296026	-0.45%
6	0.226249	0.230764	+2.00%
7	0.166617	0.206710	+24.06%
8	0.156857	0.168523	+7.44%
9	0.115062	0.133546	+16.06%
10	0.103062	0.108760	+5.53%
11	0.062057	0.074906	+20.70%
12	0.055158	0.061469	+11.44%

The first six periods agree within 2.00%. Higher Ritz modes are more sensitive to the generated load-dependent subspace: mode 7 differs by 24.06%. The report therefore distinguishes low-order period agreement and aggregate mass capture from mode-by-mode equivalence throughout the twelve-vector set.

7.4. Model 2 horizontal mass participation

The ETABS screenshot and the verified FEAKalc returned mode shapes give the following horizontal participation. FEAKalc values below are a numerical results table, calculated directly from the verified production mode shapes and diagonal mass matrix.

ETABS

Units: As Noted Hidden Columns: No Sort: None			Modal Participating Mass Ratios									
Filter: None												
	Case	Mode	Period sec	UX	UY	UZ	SumUX	SumUY	SumUZ	RX	RY	
▶	Modal	1	1.395	0.0015	0.5466	0	0.0015	0.5466	0	0.4025	0.0023	
	Modal	2	0.936	0.6117	0.0012	0	0.6132	0.5479	0	0.0009	0.3819	
	Modal	3	0.692	0.0003	0.0611	0	0.6135	0.609	0	0.0022	0.0058	
	Modal	4	0.354	9.606E-07	0.1901	0	0.6135	0.7991	0	0.2078	0.0005	
	Modal	5	0.297	0.0122	0.0108	0	0.6257	0.8099	0	0.0076	0.0152	
	Modal	6	0.226	0.2196	0.0008	0	0.8454	0.8108	0	0.0011	0.2643	
	Modal	7	0.167	0.0003	0.0339	0	0.8456	0.8446	0	0.0525	0.0007	
	Modal	8	0.157	0.0028	0.0144	0	0.8484	0.859	0	0.022	0.0047	
	Modal	9	0.115	0.003	0.055	0	0.8514	0.914	0	0.1031	0.0058	
	Modal	10	0.103	0.0702	0.0028	0	0.9216	0.9168	0	0.0055	0.1311	
	Modal	11	0.062	0.003	0.0619	0	0.9246	0.9788	0	0.1401	0.0064	
	Modal	12	0.055	0.0601	0.0021	0	0.9847	0.9808	0	0.0048	0.1406	

Figure 12. Model 2 ETABS participating-mass output for twelve Ritz vectors.

FEAKalc verified numerical results

Mode	UX	UY	Sum UX	Sum UY
1	0.0016	0.5450	0.0016	0.5450
2	0.6057	0.0014	0.6073	0.5463
3	0.0003	0.0606	0.6075	0.6069
4	0.0000	0.1900	0.6075	0.7969
5	0.0134	0.0103	0.6210	0.8072
6	0.2059	0.0011	0.8268	0.8083
7	0.0141	0.0002	0.8409	0.8085
8	0.0001	0.0355	0.8410	0.8440
9	0.0063	0.0462	0.8474	0.8902
10	0.0588	0.0068	0.9062	0.8970
11	0.0114	0.0700	0.9176	0.9670
12	0.0626	0.0071	0.9802	0.9741

At twelve vectors, FEAKalc captures 98.02% X and 97.41% Y, compared with 98.47% X and 98.08% Y in ETABS. Differences are -0.45 and -0.67 percentage points. A separately generated six-vector FEAKalc run captures 91.01% X and 85.31% Y; twelve vectors provide materially better coverage in this model.

8. Numerical verification and code scrutiny

The investigation checked the formulation and implementation behind the matching periods. The final results were obtained with the deployed production assembly, and the checks below provide evidence independent of visual similarity.

8.1. Mass and stiffness reconciliation

Horizontal mass in each direction	ETABS kg	FEAKalc kg	Difference
Model 1	4,412,804.77	4,413,028.94	+0.0051%
Model 2	13,716,256.47	13,710,890.33	-0.0391%

A fresh check on the final build produced bit-identical assembled stiffness and mass matrices when each benchmark was prepared with the static or modal analysis flag. Quad and triangular shell patch tests recovered the analytical in-plane shear energy GtA within a relative tolerance of 10^{-10} . There is no additional modal-only shell shear stiffness in the tested configuration.

Mesh interfaces, base-support inheritance and column to slab coupling were reviewed explicitly. The approved column connection penalty scale of 0.0001 is shared by static and modal assembly. Support inheritance applies translational restraints along qualifying supported wall-base edges; it does not silently add rotational fixity. These are part of the stated benchmark idealisation.

8.2. Independent eigen solution

An independent SciPy sparse eigen solution used FEAKalc exported K and M matrices, including the kN to N conversion. It gave fundamental periods of 2.102558468 s and 1.382459323 s, agreeing with the final twelve-vector Ritz fundamentals to better than $10^{-8}\%$ in both cases. This verifies extraction for the assembled system; the independent calculation is not a second physical model or a run of FEAKalc's separate built-in eigen branch.

8.3. Ritz implementation and regression checks

Code review identified two avoidable losses of mass capture: completing one directional sequence before starting the other, and generating extra vectors whose higher reduced modes were discarded. Generation now cycles through the selected starting loads, maintains separate inertia-load sequences, and retains the requested independent basis. Two passes of mass orthogonalisation control loss of independence.

The numerical suite passed 56 checks covering PCG preparation equivalence, one-vector and odd/even requests, optional vertical excitation, missing starting loads, true basis exhaustion and mass orthogonality. An analytical complete-subspace example recovered the exact eigenfrequencies and 100% participating mass. Full runs on both benchmarks gave modal mass-orthogonality errors below 1.3×10^{-14} .

The corrected Ritz runs preserved geometry, supports, couplings and mass exactly. Stiffness was unchanged within assembly roundoff, with a maximum relative matrix difference of 7.7×10^{-17} . The improvement in captured mass therefore comes from the modal basis correction rather than an adjustment of the physical model.

8.4. Conclusions and reproducibility

These two benchmarks demonstrate close agreement between FEAKalc and ETABS in fundamental periods, the first six periods from twelve-vector Ritz runs, and cumulative horizontal participating mass. The independently checked stiffness and mass assembly, analytical shell tests, sparse eigen solution and regression suite support the numerical credibility of the comparison.

The agreement applies to the stated linear elastic models, mass sources, support assumptions, connection idealisation and 1.25 m discretisation. Different shell element layouts and Ritz subspaces remain. Higher numbered Ritz modes, vertical excitation response, nonlinear or cracked-concrete response, response-spectrum member forces and general mesh convergence are outside the established conclusions. No quantitative spatial mode-shape correlation has been calculated across the two meshes.

9. Technical references

[R1] Computers and Structures Inc. [Modal Case Data Form](#). Definition of modal case types, starting loads and generation cycles. Accessed 25 October 2026.

[R2] Computers and Structures Inc. [Ritz vs Eigen vectors](#). Load-dependent Ritz vectors and their participation efficiency. Accessed 25 October 2026.

[R3] Computers and Structures Inc. [Ritz vectors](#). Dependence of higher Ritz modes on the requested basis size. Accessed 25 October 2026.